Chapter 7: Inductors for High Voltage Power Supplies

As we have said in the previous chapter, magnetic components are very important in the field of power conversion. Because this book deals with high voltage power supplies, we will start with the basic inductor and present its key features. In this chapter we will learn how to design inductors within the constraints of permeability shifts, DC and AC working currents, material saturation, and temperature rise. This will pave the way for more complicated devices in the chapters ahead such as the high voltage transformer (Chapter 8) and driver circuits (Chapters 9+).

Inductors are usually found in high voltage power supplies. From the 60Hz variac/transformer step-up types that use lamination chokes to smooth out the ripple to high frequency resonant converters that tout low stored energy within their topology, the following are the usual locations detail where inductors are found. We will cover them in order.

1. Tuned Circuits (air core)
2. Ringing Choke Converters
3. Input line chokes
4. Output filter chokes
5. Resonant power chokes

But first a review of the equations that represent the inductor:

Basic Inductor:
The basic inductor equation from elementary circuit theory relates the voltage across an inductor to the change in current flowing through it is:

$$V(t) = L\, di(t)/dt \qquad (7\text{-}1)$$

Conversely, the integral of the voltage function will yield the current:

$$i(t) = 1/L \int v(t)\, dt \qquad (7\text{-}2)$$

When only a DC current flows through an inductor the voltage across it is from Ohmic losses only and these are due to the resistance of the copper wire and terminations. But when there is an AC steady state current flowing through the inductor, the following equation, really an AC form of Ohm's Law, comes in handy:

$$V_L = I_L\, Z \qquad (7\text{-}3)$$

which states that the steady state AC voltage across an inductor depends upon the AC steady state current flowing through, I_L, it multiplied by the impedance of the inductor Z. If the units of I_L are Amperes RMS, the units of the voltage, V_L, will be volts RMS as well. Of course, the units of the impedance Z are Ohms. But Z is frequency sensitive. In phasor notation the impedance of an inductor is given by:

$$Z = j\omega L = j2\pi fL \qquad (7\text{-}4)$$

with the operator j, indicating a 90° phase shift in time between the voltage and current, with the voltage leading the current as remembered by the mnemonic: *Eli the ice man*. As you can see, for any inductor, the higher the frequency the higher the impedance. Phasor notation, brought to general use by Heaviside and Steinmetz in the 1900's, makes analyzing AC circuits easier because it converts lengthy differential equations into easier algebraic ones. Laplace Transforms do the same but are a little more complicated because they offer the transient solutions as well.

As you probably know, the basic definition of inductance is given as the first derivative of the magnetic flux with respect to current. That is:

$$L = d\phi \,/\, di \qquad (7\text{-}5)$$

The units of inductance are the Henry named after the American electrical scientist of the 1800's. Most inductors fall into the range of 1 μH up to 10H covering seven orders of magnitude. Even a straight piece of wire has an inductance of about 1.5 μH/meter.

An inductor can store energy in the magnetic field it based on the instantaneous value of current:

$$Energy = \tfrac{1}{2} Li^2 \qquad (7\text{-}6)$$

When the current is in Amperes, inductance in Henrys, the energy is computed in Joules.

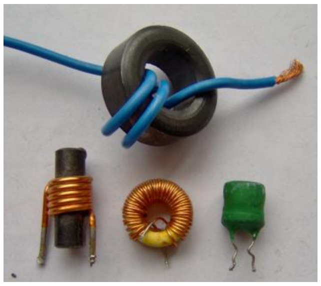


Figure 7.1: Basic Inductors (Miguel)

Calculation of Inductance of an Air-Core Solenoid

There are many configurations that one can utilize to wind an inductor and several are listed here. Unfortunately, calculating the inductance of a winding is not as easy as one might expect. Although we developed equations in Chapter 5 that gave us the B field as a function of current and position within a solenoid of length L of radius R, it did so at locations *only along the central axis* at a distance from the center of the solenoid z:

$$B_z = \frac{\mu_0 N I}{2}\left(\frac{l/2 - z}{l\sqrt{R^2 + (l/2 - z)^2}} + \frac{l/2 + z}{l\sqrt{R^2 + (l/2 + z)^2}}\right). \qquad (7\text{-}7)$$

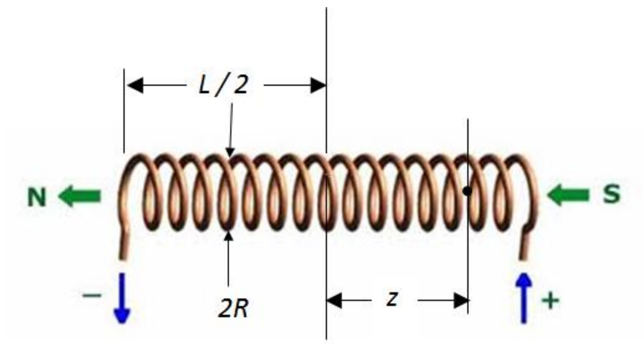


Figure 7.2: Long solenoid inductor

No mention was made of the B field off the central axis, say closer to the coil edge where it stands to reason that the B field would be obviously different. Nevertheless, we refined the above equation to consider the spot where z = 0, at the exact center of the coil. This yielded a B field:

$$B \quad = \quad 0.5\, \mu_o\ N\, i \,/\, SQRT\,(R^2 + \ell^2/4) \qquad (7\text{-}8)$$

The flux can be found by making the assumption that the B field is constant at this value from the central axis to the wire coil, that is, across the surface area of the circle or coil diameter. It's a poor postulation but seems to work. The flux can be found by remembering:

$$\Phi \quad = \quad \int B \cdot dA \qquad (7\text{-}9)$$

and simply multiplying by the coil crossectional area we get the magnetic flux:

$$\Phi \quad = \quad 0.5\, \mu_o\ N\, i\, \pi R^2 / \,SQRT\,(R^2 + \ell^2/4) \qquad (7\text{-}10)$$

This allows us to get the inductance for an air-core solenoid:

$$L \quad = \quad N d\Phi / di \quad = \quad 0.5\,\mu_o\, N^2 \pi R^2 / SQRT\,(R^2 + \ell^2/4) \qquad (7\text{-}11)$$

Let's try an example:

Example 1:
Calculate the inductance of a single layer air-core solenoid wound on a paper towel tube of diameter 1.75 inches and a winding length of 3.5 inches. The solenoid is wound with 108 turns of #35 AWG red magnet wire. This inductor will be used in conjunction with a 365 pF variable capacitor, as part of a tuning circuit of a young person's crystal radio project. The turns are wound snug next to each other.

Solution:
Converting our units to MKS:

ℓ = 3.5 inches (0.08890 meters)
D = 1.75 inch (0.04445 meters)
N = 108

therefore the radius of the coil form R is:

R = 0.875 inches (0.02225 meters)

Using our derived equation we can easily plug in the numbers:

$$L = 0.5\,\mu_o\, N^2 \pi R^2 / SQRT\,(R^2 + \ell^2/4)$$
$$L = (0.5)(4\pi E\text{-}7)(108)^2 \pi (0.02225)^2 / SQRT((0.02225)^2 + (0.0889)^2/4)$$
$$L = 2.943E\text{-}6 / 0.04971$$
$$L = 229.3\ \mu H$$

This inductance when utilized with the tuning capacitor mentioned will allow the range from 550 kHz to 1.65 MHz be received. But that equation is rather complicated. What about our shortened form for B field we generated in Chapter 5, what will that give? Using this "short" B field equation we get (using meters as our length):

$$B = \mu_o\, N\, I / \ell$$
$$\Phi = \mu_o\, N\, I\, A / \ell$$
$$L = N\, d\Phi / di \quad = \quad \mu_o N^2 A / \ell \quad = \quad \mu_o N^2 \pi R^2 / \ell \qquad (7\text{-}12)$$
$$L = 256.4\ \mu H$$

Close, but which one is correct? That is a good question. There are a multitude of inductor equations that have been developed since 1900. Here are three of the literally twenty equations one can find online dealing with the inductance of a single layer solenoid coil.

Wheelers equation: $L = N^2R^2 / (9R + 10\ell)$ *uses inches gives μH* (7-13)
$L = 208.3\ \mu H$

Esnault-Pelterie: $L = 0.1008\ (R^2N^2)/(\ell + 0.92R)$ *uses inches gives μH* (7-14)
$L = 218.4\ \mu H$

ARRL handbook $L = D^2N^2/(18D+40\ell)$ in inches gives μH (7-15)
$L = 208.3\ \mu H$

The ARRL handbook equation gives the same inductance as the Wheeler formula because it is really the same equation just corrected for a diameter. But, if you get tired of calculating the inductance there are also automatic air-core inductance calculators online and this is what they yield when our data is inserted:

www.allaboutcircuits.com/tools/coil-inductance-calculator	L	=	265 μH
www.66pacific.com/calculators/coil-inductance-calculator.aspx	L	=	216 uH
www.translatorscafe.com/unit-converter/en-US/calculator/coil-inductance/?D=2&Du=cm&l=1&lu=cm&N=10	L	=	222 μH
https://hamwaves.com/inductance/en/index.html#input	L	=	219 μH

As you can see there is no consensus on what the exact inductance would be in this situation even with online "calculator" sites. They all yield different values although they are reasonably close enough for the beginners project of a crystal radio. If you are going to need an exact inductance value the old method of "cut and try" cannot be underestimated.

Air core inductors made with a single layer coil has two advantages. Firstly, like all air core coils, it is free from 'iron losses' and the non-linearity of permeability found in a BH curve. Secondly, single layer coils have the additional advantage of low self-capacitance and thus high self-resonant frequency. These coils are mostly used for RF work. You will find that many first crystal radios that are built utilize inductors wound on paper towel tubes.

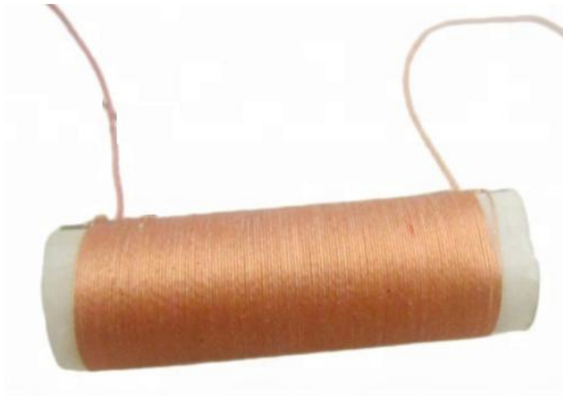


Figure 7.3: Air-core paper-towel tube solenoid

Because they come up so frequently, we will list the inductance of a toroid shape:

$$L = (\mu_o h N^2 (\ln (b/a)) / 2 \pi) \quad (7\text{-}16)$$

where h is the thickness of the toroid and b and a the outer and inner diameters. Like the linear air-core solenoid, the toroid has many online equations. Here is one that forgoes the natural log function:

$$L = 0.01257N^2(R - SQRT(R^2-r^2)) \quad (7\text{-}17)$$

where R is the average radius of the core and "r" the radius of the toroid cross-section circular shape – that is - the radius of the turn made by the wire so to speak. This gives the inductance in uH if the dimensions listed are in cm.

Figure 7.4: Toroidal Inductor

Permeability

In Chapter 6 we examined the increase in magnetic field when a ferromagnetic material is inserted within the windings of an air-coil inductor. Consider an air-core solenoid where the inductance is given by the following (we will use the short version) equation:

$$L = \mu o\, N^2 A / \ell \quad \text{(short form air core inductor R<< L)} \qquad (7\text{-}18)$$

Upon inserting a ferromagnetic material we will find that the inductance has now increased sometimes by a substantial amount. Multiplying by the relative permeability of the material gives us the increased inductance of the solenoid wound on a magnetic material:

$$L = \mu_r\, \mu o\, N^2 A / \ell \qquad (7\text{-}19)$$

We call μ_r the *relative* permeability of the material, a number that can run from 1.0 to 100,000. Obviously this is the easy way of looking at the situation. Exact solutions require one to consider the actual placement of the material position inside the solenoid, the size of the core, the magnetic path length, etc. Placing a BB sized piece of ferrite inside our much larger air-core solenoid of Example 1 will not increase the inductance by very much and all of these parameters have to be taken into consideration. As mentioned in Chapter 6, the relative permeability μ_r, can be obtained from the B-H curve of the material. Figure 7.5 shows the following relationship between μ_r and the slope:

$$\mu_r = (1/\mu_o)\, dB/dH \qquad (7\text{-}20)$$

Notice that the relative permeability is very sensitive to the H field setup within the material. As the H field excitation of the inductor starts out small, say for signal magnitudes, the permeability of the material begins at a low value called the *initial permeability*, μ_i. As the H field increases a maximum point is reached, μ_m. Past this maximum we have a decreasing relative permeability that eventually winds up heading towards unity when saturation is reached.

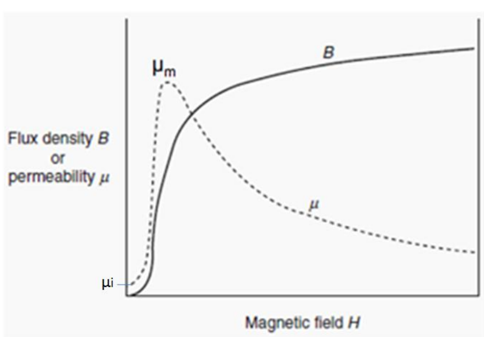


Figure 7.5: Permeability vs H field

When designing inductors you will see many different permeability terms bandied about prompting the question: which one should we use to calculate inductance? The best answer always is to use what is found in a vendor's catalog. They are the ones making the material so they should be on top of the numbers for their products. One parameter that usually comes up often is the A_L value for the particular core you have selected. Although not a permeability number, the A_L gives the inductance for a certain listed number of turns, usually just one turn. This allows easy calculation of inductance. Usually listed as nano-Henry per turn squared, you can find A_L values listed in Figure 7-6 for the Ferroxcube 1811 pot core (2013 catalog).

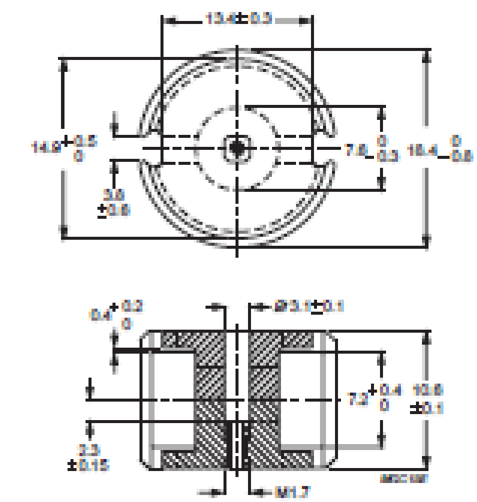
Ferroxcube

P cores and accessories — P18/11

CORE SETS

Effective core parameters

SYMBOL	PARAMETER	VALUE	UNIT
$\Sigma(I/A)$	core factor (C1)	0.597	mm^{-1}
V_e	effective volume	1120	mm^3
I_e	effective length	25.8	mm
A_e	effective area	43.3	mm^2
A_{min}	minimum area	36.0	mm^2
m	mass of set	≈6.0	g



Dimensions in mm.

Fig.1 P18/11 core set.

Core sets for filter applications

Clamping force for A_L measurements, 80 ±20 N.

GRADE	A_L (nH)	μ_e	TOTAL AIR GAP (μm)	TYPE NUMBER (WITH NUT)	TYPE NUMBER (WITHOUT NUT)
3D3	63 ±3%	≈ 30	≈ 1210	P18/11-3D3-E63/N	P18/11-3D3-E63
	100 ±3%	≈ 47	≈ 670	P18/11-3D3-E100/N	P18/11-3D3-E100
	160 ±3%	≈ 76	≈ 370	P18/11-3D3-E160/N	P18/11-3D3-E160
	1400 ±25%	≈ 665	≈ 0	–	P18/11-3D3
3H3	160 ±3%	≈ 76	≈ 400	P18/11-3H3-E160/N	P18/11-3H3-E160
	250 ±3%	≈ 119	≈ 240	P18/11-3H3-A250/N	P18/11-3H3-A250
	315 ±3%	≈ 149	≈ 180	P18/11-3H3-A315/N	P18/11-3H3-A315
	400 ±3%	≈ 190	≈ 140	P18/11-3H3-A400/N	P18/11-3H3-A400
	630 ±5%	≈ 299	≈ 80	P18/11-3H3-A630/N	P18/11-3H3-A630
	3100 ±25%	≈ 1470	≈ 0	–	P18/11-3H3

Figure 7.6: Information about an 1811 pot core (Ferroxcube)

The next important figure describes the material, in this case 3C81, a ferrite normally used to make power transformers and inductors. On this datasheet there are charts that show how the material behaves over temperature and driving conditions. Notice for example in Figure 7-7 (Fig 2) how the permeability drops off suddenly to zero at the Curie Point of the material of 230 °Celsius. All magnetic materials have data sheets that tell their properties.

Ferroxcube

Material specification 3C81

3C81 SPECIFICATIONS

A low frequency power material with minimum power losses around 60 °C for use in power and general purpose transformers at frequencies up to 0.2 MHz.

SYMBOL	CONDITIONS	VALUE	UNIT
μ_i	25 °C; ≤10 kHz; 0.25 mT	2700 ±20%	
μ_a	100 °C; 25 kHz; 200 mT	5500 ±20%	
B	25 °C; 10 kHz; 1200 A/m	≈ 450	mT
	100 °C; 10 kHz; 1200 A/m	≈ 360	
P_V	100 °C; 25 kHz; 200 mT	≤ 185	kW/m³
ρ	DC; 25 °C	≈ 1	Ωm
T_C		≥ 210	°C
density		≈ 4800	kg/m³

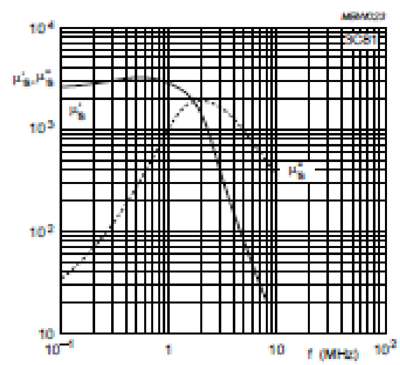


Fig.1 Complex permeability as a function of frequency.

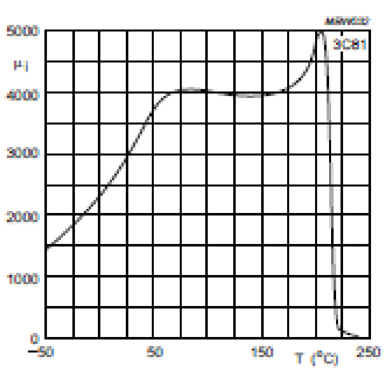


Fig.2 Initial permeability as a function of temperature.

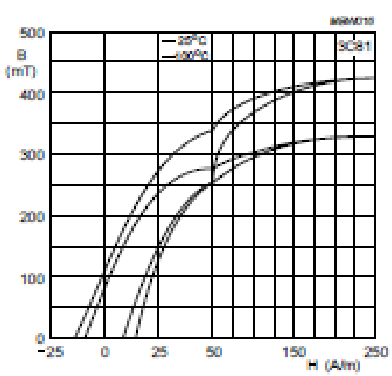


Fig.3 Typical B-H loops.

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Figure 7.7: Material 3C81 data sheet

A word of caution. Make sure you know exactly the units being used for the A_L calculation. For example, sometimes a vendor will publish the A_L values for a particular core. This gives the inductance value as a function of the number of turns. Old Ferroxcube catalogs from the 1970's would specify A_L in mH/1000 turns. Newer ones list nH/turn-squared. When you work it out, they give the same value, that is 45 nH/T^2 = 45 mH/1000 turns.

Example 2:
A pot core inductor of size 1811 is made from 3D3 material without a gap. The A_L value of this is seen to be 1400 nH/$turn^2$. How many turns are required to make a 1.0 mH inductor?

Solution:
The A_L equation works with the square of the turns:

$$L = A_L N^2 \quad (7\text{-}21)$$

where A_L = 1400E-9 Henry/$turn^2$ (from the datasheet). For a 0.001 Henry choke to be made we solve the equation:

$$N = SQRT\ (L/A_L)$$
$$N = SQRT\ (0.001\ /\ 1400E\text{-}9)$$
$$N = 26.7 \text{ turns (round up to 27 turns)}$$

Ferrite core analysis:
Many DC to DC converters utilize ferrite cores for their power conversion topology. There are many reasons to do so, among the better ones is that over the last twenty years ferrites have been designed to operate at frequencies much greater than 1Mhz – something tape wound Permalloy or powdered iron cores simply cannot do. In addition, some ferrite cores such as pot cores or RM types can be assembled with bobbins wound with many turns of fine wire and easily put together. This is not the case with a toroidal core which has to be wound on a special machine. This is especially concerning for high voltage step-up converters which sometimes rely on thousands of turns to make a transformer secondary winding. It is very hard to wind a toroid with a thousand turns carefully insulated from each other. As a rule, most high voltage transformers are made using ferrites that are not toroids.

In addition, a ferrite core made from two pieces can have its inductance *adjusted* to a certain value by placing an insulating gap between the two core halves. This cannot be done with a toroid core. We will take a closer look at this ability now because we will use it when we design inductors for ringing choke power supplies.

Effective Permeability

Here is the detailed listing for an 1811 pot-core set from the Ferroxcube catalog. Looking at the A_L value for a core made of 3C81 material shows a value of 4000 nH/T^2 for an inductor made without a gap. They also show an "effective permeability" of 1900. This seems odd considering the 3C81 material has an initial permeability of 2700 as given in its materials property sheet of Figure 7-7.

GRADE	A_L (nH)	μ_e	AIR GAP (µm)	TYPE NUMBER
3C81	100 ±3%	≈ 47	≈ 710	P18/11-3C81-E100
	160 ±3%	≈ 76	≈ 400	P18/11-3C81-A160
	250 ±3%	≈ 119	≈ 240	P18/11-3C81-A250
	315 ±3%	≈ 149	≈ 180	P18/11-3C81-A315
	400 ±3%	≈ 190	≈ 140	P18/11-3C81-A400
	4000 ±25%	≈ 1900	≈ 0	P18/11-3C81

Figure 7.8: A_L values for 1811 core 3C81 material

The answer to this confusing observation is that Ferroxcube insures the inductor you calculate using the ungapped A_L listing will have the same value if you calculate it using the inductance equation for a solenoid providing you utilize this "effective permeability" shown here as 1900. Somewhere over the years at Ferroxcube a technician made a direct measurement made on an 1811 core with a bobbin containing a certain number of turns and these values of A_L were recorded for the catalog. By posting the effective permeability, they have taken the guesswork out of what permeability to use. We will use this number when gapping cores later on.

As proof that the "short" equation works for ungapped cores consider an ungapped 1811 core (P18/11-3C81) wound with 100 turns. The A_L value for the 3C81 material indicates we will have an inductance of:

$$L = A_L T^2$$
$$L = (4000E\text{-}9)(100)^2$$
$$L = 0.04 \text{ Henry}$$

Now, consider the "short" inductance equation for a solenoid using the "effective permeability":

$$L = \mu_r \mu o N^2 Ae / \ell e$$
$$L = (1900)(4\pi E\text{-}7)(100^2)(0.433E\text{-}4)/(0.0258)$$
$$L = 0.04 \text{ Henry}$$

where the core area and magnetic path length, obtained from the catalog, have been inserted, all in meters. From this expression we get the same value providing we use this number listed.

Gapping

One of the beauties of using cores with two halves is that you can insert a gap when needed to adjust the inductance, prevent DC saturation or store more energy when needed. An air gap not only lowers the "effective permeability" of the system but increases the effective magnetic path length as well. Air gaps have other purposes too - such as reducing the percentage drift in inductance over temperature.

Many ferrite cores can be purchased with their center leg already gapped (shorter). While this may sound good for large production runs, and the shielding does prevent the magnetic field from radiating, it is probably not a good idea practical reasons. First of all, doing so would require you to stock a ferrite by its gap. What will happen if you need to change the inductance because of a modification down the line? It's hard to add a gap to an already gapped ferrite. The second problem with a gapped ferrite where the center leg is shaved down is susceptible to cracking when tightening the center mounting screw. Avoid pre-gapped cores because, as times go on, they may also be hard to procure.

Example 3:

Using the Ferroxcube datasheet for the 1811 pot core, determine the effective gapped permeability for a core set that has a *total* gap of 500 μm (0.02 inches) made with 3C81 material. This is accomplished by placing a gaping material of 10 mils thickness between the pot core halves. In addition, determine the inductance of a core wound with 100 turns having this gap.

Solution:

The gap length of 500 μm is not included on the Ferroxcube catalog page (Figure 7.8). The closest value to the one we need is 400 μm. Using EXCEL and TRENDLINE analysis we can plot the gapped effective permeability μ_e and A_L value for a continuous range of gaps taken from their catalog. This is shown in Figure 7.9. The position of 500μm is shown by the dotted red line. Here an A_L value can be calculated as 133.5 nH/T^2. From this we can determine the inductance of a core set having that gap and the 100 turns. Remember, the values for the gap in the core represent the total gap length. Because for pot cores only the center leg is gapped (cut short), the gap value represents how much shorter the central leg is.

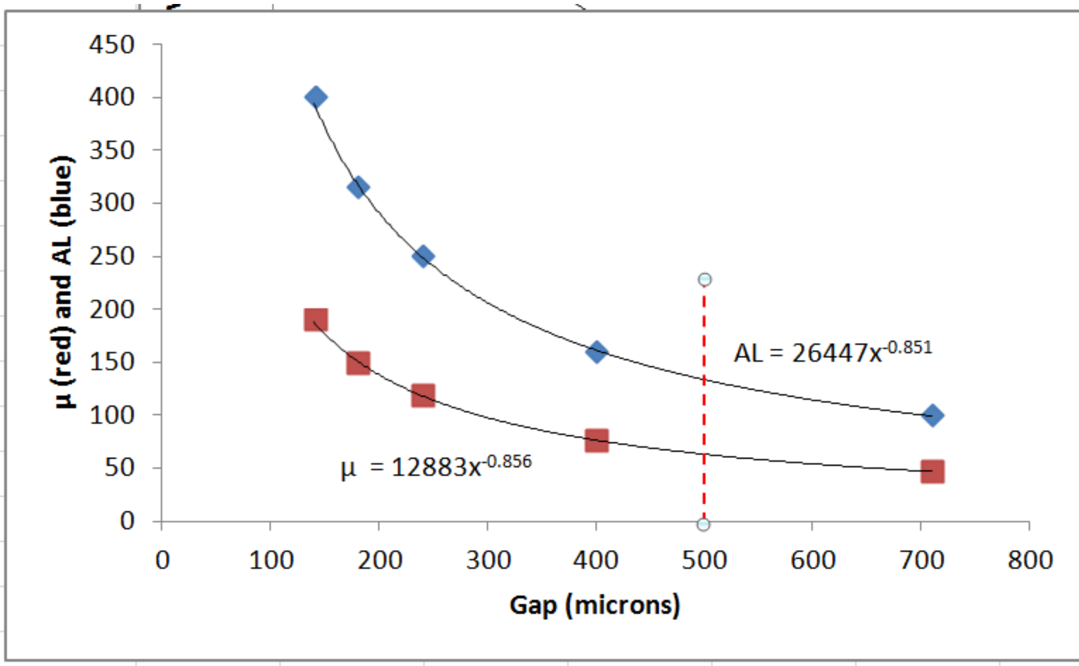


Figure 7.9: Effective Gap Permeability and A_L as a function of gap using data points from the catalog

The A_L values as a function of gapping show the following TRENDLINE:

$$A_L = 26447\,\ell gap^{-0.851}$$

The gapped effective permeability show this TRENDLINE:

$$\mu_{e\ gapped} = 12883\ \ell gap^{-0.856}$$

where ℓgap is in micrometers. Because we need the values at ℓgap = 500 μm, inserting the value of 500 into both equations yields:

$$A_L = 133.5\ nH/T^2 \quad \text{at a 500 μm gap}$$

$$\mu_{e\ gapped} = 63 \quad \text{at a 500 μm gap}$$

The inductance for this core with 100 turns would be:

$$L = A_L N^2$$

$$L = (133.5E\text{-}9)(100)^2$$

$$L = 0.00134\ Henrys$$

If we used the "short" inductance equation with this value of effective gapped permeability we should get the same value of inductance:

$$L = \mu_{e\ gapped}\, \mu o\, N^2\, Ae \,/\, \ell e \qquad (7\text{-}22)$$

$$L = (63)(4\pi E\text{-}7)(100^2)(0.433E\text{-}4)/(0.0258)$$

$$L = 0.00133 \text{ Henrys}$$

where both Ae and ℓe are the 1811 core area and magnetic path length, both values are taken from the catalog (in meters). You can see that knowing the "effective" gapped permeability $\mu_{e\ gapped}$ sometimes is useful. Plotting the parameters taken from actual core samples as listed in the catalog is the best way to obtain values for gaps not listed. But what if the catalog doesn't show values for gapped cores?

Gapped Core Equation

We will derive a formula that gives us the inductance for a set of cores that can be separated using a gap by using energy relationships. You can use this if you cannot obtain information on gapped cores as we did in example 3. This is called the "universal gapping equation" and can be used anywhere a core is to have a gap placed in it, working for both metglass cores, laminations and ferrites. To make it easier we will start off with an inductor composed of two halves with a small gap inserted in between and driven by a sinewave. The length ℓe is the magnetic material length as purchased and ℓgap is the total gap you are adding. Ae is the cross-sectional area of the core.

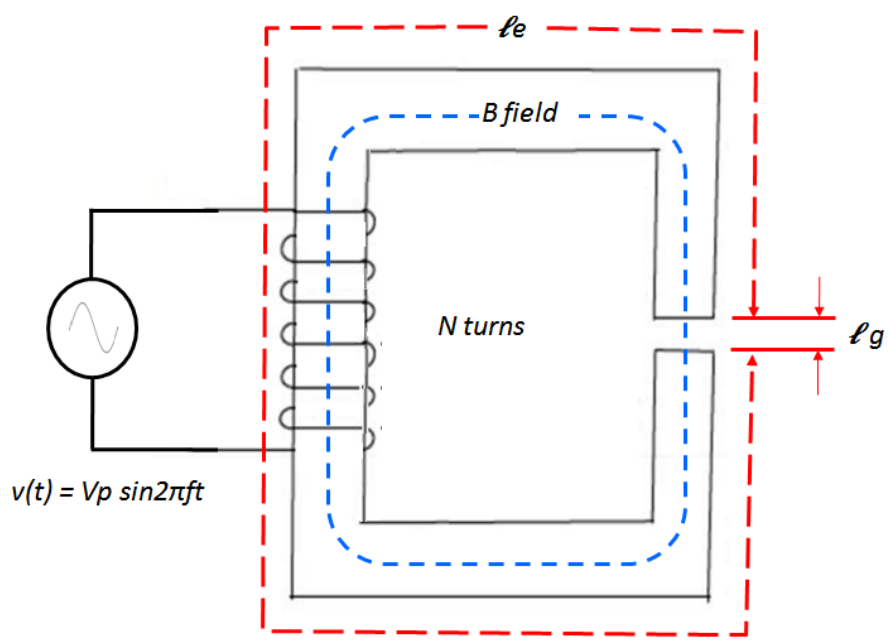


Figure 7.10: Ferrite core of physical length ℓe with gap ℓg

Starting off with the inductor equation:

$$i(t) = 1/L \int v(t)\,dt = 1/L \int Vp \sin 2\pi ft\,dt$$

we can easily determine the current waveform. It will be sinusoidal as well:

$$i(t) = -(Vp/2\pi fL) \cos 2\pi ft + C \qquad (7\text{-}23)$$

where, because the DC level is zero, C = 0.

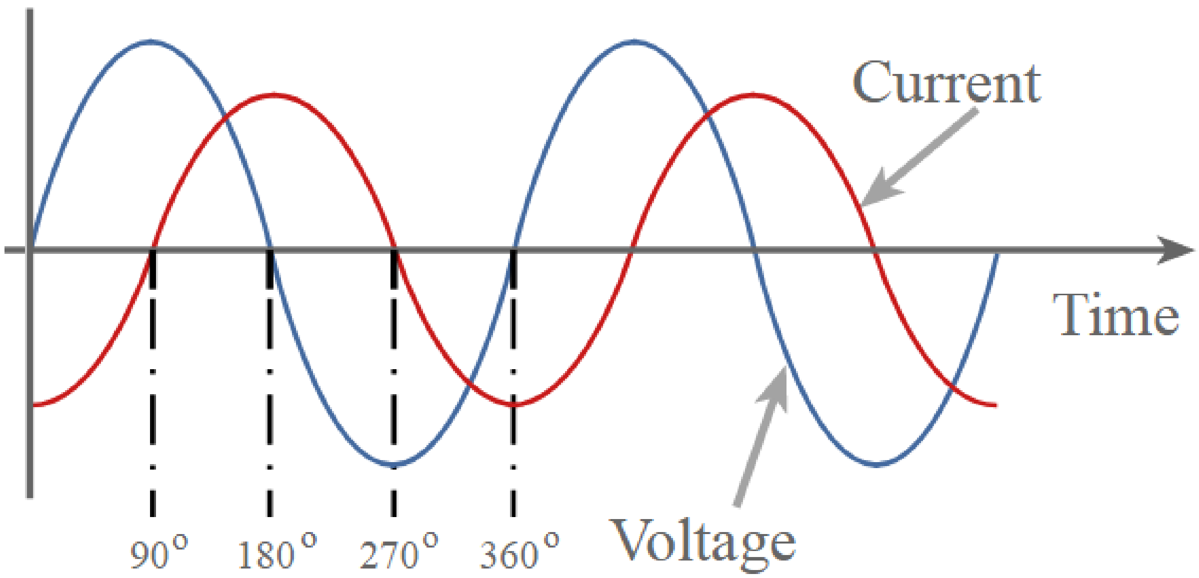


Figure 7.11: Voltage and resultant current waveforms (electronics-notes.com)

The power going into the circuit is simply the product of voltage and current, it is not zero because there are times when both voltage and current overlap with finite values:

$$P(t) = -(Vp^2 / 2\pi fL) \sin 2\pi ft \cos 2\pi ft \qquad (7\text{-}24)$$

and the energy is just the integral of the power:

$$Energy\,(t) = \int P(t)\,dt = -(Vp^2 / 2\pi fL) \int \sin 2\pi ft \cos 2\pi ft\,dt \qquad (7\text{-}25)$$

because $$\int \sin ax \cos ax \; dx = (1/2a) \sin^2 ax \qquad (7\text{-}26)$$

we find the *maximum* energy into the system due to the electrical excitation as:

$$Energy_{maximum} = Vp^2 / 8\pi^2 f^2 L \qquad (7\text{-}27)$$

and this energy from the generator will manifest itself as magnetic field energy.

As we learned from the chapter on magnetism, Chapter 4, the energy of a magnetic field per unit volume depends upon the square of the B field:

$$Energy = ½ B^2 / \mu_0 \qquad (7\text{-}28)$$

These energy relationships have to be equal because the electrical voltage from the source is driving the inductance and using current causing the magnetic field to exist. Current flows into the coil as the magnetic field grows. When the voltage starts dropping, the magnetic field energy flows out of the winding back to the generator voltage source in a lossless cycle.

By traveling around the magnetic path and summing up the volumes where the magnetic field exists in both the magnetic material and the air-space gap, we can determine the inductance of the gapped core, taking into consideration several factors:

1. The B field magnitude in a closed loop is constant in value. This is in the magnetic material as well as the air-gap, both regions have the same B value.
2. If a gap is added to a core system, the volume added is: $Ae \; \ell gap$
3. Equation 7-28 must include the relative permeability when necessary.

Which, when incorporated yields the following energy summation over the full volume:

$$Energy = ½ B^2 Ae \; \ell e \; / \mu_r \mu_0 + ½ B^2 Ae \; \ell gap \; / \mu_0 \qquad (7\text{-}29)$$

where Ae is the crossectional area of the core and ℓe the magnetic path length of the core. summing up we have:

$$\text{Energy} = ½ B^2 Ae \, (1/\mu_0) (\ell e / \mu_r + \ell gap) \qquad (7\text{-}30)$$

If we can relate the continuous B field to the voltage impressed on the coil of the inductor we could determine the inductance of the gapped inductor.

Faraday's Law teaches us that the time derivative of the magnetic flux is proportional to the voltage of an inductor:

$$v(t) = N\, d\Phi / dt$$

and we know the magnetic flux over an area of constant B field is just"

$$\Phi = \int B \cdot dA$$

yielding:

$$v(t) = NAe\, dB / dt$$

Becaue *Ae* is the area that the B field flows through. Because we defined the driving voltage as:

$$v(t) = Vp \sin 2\pi ft$$

we can easily find the B field maximum as a function of time:

$$B(t) = (1/NAe) \int v(t)dt = (1/NAe) \int Vp \sin 2\pi ft\, dt \qquad (7\text{-}31)$$

giving us:

$$Bmax = Vp / NAe2\pi f \qquad (7\text{-}32)$$

Inserting this into equation (7-30) we find the magnetic energy as:

$$\text{Energy}_{\text{max}} = \tfrac{1}{2} (Vp / NAe2\pi f)^2\ Ae\, (1/\mu_0)\, (\ell e / \mu_r + \ell gap) \qquad (7\text{-}33)$$

but this is equal to the energy term we derived for the applied electrical energy, equation (7-27), with the result that the inductance of the gapped core can now be found:

$$L_{gapped} = N^2 Ae\, \mu_0 \mu_r / \ell e\, (1 + \mu r\, \ell gap / \ell e) \qquad (7\text{-}34)$$

Notice that the inductance is just the inductance of the un-gapped core divided by the factor involving the core length and gap length and relative permeability of the core:

$$L_{gapped} = L_{ungapped\ core} / \ (1 + \mu r\, \ell gap / \ell e) \qquad (7\text{-}35)$$

Let's test this out.

Example 4:
Using the fact that an ungapped 1811 3C81 ferrite core of 100 turns has an inductance (according to the A_L of the catalog of 4,000 nH/T^2) of 0.04 H, determine the inductance of a core with the same number of turns gapped with a *total* gap of 500 μm (0.02 inches).

Solution:
Obtaining the ferrite length, and effective permeability we can insert this information into equation 7-38:

$$L_{gapped} = L_{ungapped\ core} / (1 + \mu r\, \ell gap / \ell e)$$

$$L_{gapped} = 0.04 / (1 + (1900)(5E\text{-}4)/(25.8E\text{-}3))$$

$$L_{gapped} = 0.04 / 37.82 \text{ Henries}$$

$$L_{gapped} = 1 \text{ mH}$$

somewhat smaller than the result we came up with in Example 3 earlier (0.00134). Why is this inductance calculated with our "universal gapping equation" smaller than that found by experiment?

Coming up short
In the literature on magnetic devices you will often see this "universal gapping equation". It shows a shortcut way to calculate the effective permeability or inductance value as a function of gap length ℓg and magnetic path length ℓe using the ungapped relative permeability μ_r, and is a consequence of our derived equation (7-35):

$$\mu_{e\ gapped} = \mu_r / (1 + \mu_r\, \ell g / \ell e))$$

In McLyman's comprehensive book *Transformer and Inductor Design Handbook* (1988) this permeability gapping equation comes up many times. Let's look at an example.

Example 5:
Using the Ferroxcube datasheet for the 1811 pot core and the "gapping equation", determine the effective gapped permeability $\mu_{e\ gapped}$ for a core set that has a *total* gap of 500 μm (0.02 inches) made with 3C81 material. We did this using EXCEL in Example 3 and found a value of 63.

Solution:

As noted earlier, the gap length of 500 μm is not included in the list of manufactured parts by Ferroxcube in Figure 7-8. Using the above equation we get (using meters):

$$\mu_{e\ gapped} = 1900\ /\ (1 + (1900)(500E\text{-}6)\ /\ (0.0258))$$

$$\mu_{e\ gapped} = 1900\ /\ (37.8)$$

$$\mu_{e\ gapped} = 50.2$$

You can immediately see the problem. We proved earlier from EXCEL plotted catalog data (taken from lab tests) that the $\mu_{e\ gapped}$ was 63. This is 25% higher than the value given by our universal "gapping equation" which is a valid magnetic equation. So which answer is correct?

The answer to this thorny problem is that the TRENDLINE analysis is the one to use. As we know, it takes into consideration the best fit curve from the data points listed for that material and these were determined experimentally. These values of A_L and $\mu_{e\ gapped}$ are larger than one would calculate using our "universal gapping equation".

The cause of this divergence is not so obvious. We have said that the values in the catalog represent actual data taken on core sets. They are the closest to the real world values one will get and should be taken as gospel. Plotting the $\mu_{e\ gapped}$ obtained from both the catalog and our "universal gapping equation" shows that the deviation gets larger with larger gap.

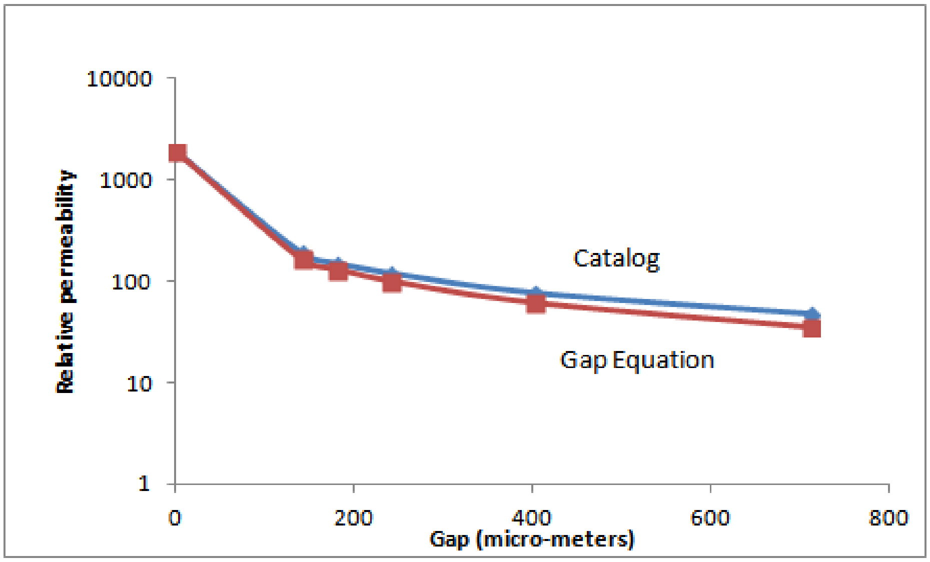


Figure 7.12: Values of effective permeability from catalog and from equation

This disagreement in permeabilities with gapping shows a smoking gun that is due to the fringing of the magnetic field between the core halves in the real world as we introduce larger and larger gaps. When the magnetic field spreads out the “effective cross-sectional area” of the core increases raising the actual *observed* inductance. Our gapping equation doesn’t take this into account. That is why the actual A_L and $\mu_{g\ effective}$ values from the catalog are always higher (except at zero gap).

At the risk of sounding repetitive, the best technique to use when working with cores that you plan to gap is to plot the data points for known values of gap from the catalog just like we did in Figure 7-9 because that data came from actual test cores and cannot be questioned. Using a TRENDLINE analysis will yield an equation that can be used to determine A_L and $\mu_{e\ gapped}$ values not found in the catalog.

For our 1811-3C81 material core the official value for a 500 μm gap is:

$$A_L = 133.5\ nH/T^2$$

and

$$\mu_{e\ gapped} = 63$$

as we determined from EXCEL. These values include the effects fringing whatever the amount is. The value of 50.2 that we found for μ DOES NOT mathematically take into consideration the fringing and should not be used if you want to generate correct inductance values. However:

For those purists who want a closed form of inductance when the fringing field *is taken into account*, the following equation, from Encyclopedia Magnetica, generates a fringing factor that one may multiply by to increase the area of the core, *Ae*, and calculate a larger inductance, A_L value or relative gapped permeability.

$$FF = 1 + (\ell g / SQRT\ Ae)(\ln (2G/\ell g)) \qquad (7\text{-}36)$$

where ℓg is the gap length in meters, *Ae* the cross-sectional area of the core in square meters, and G is a height dimension on the window of the core, that is, the cross-section of the winding area W_a. Using the values here for the 1811 pot core we find:

$$Ae = 0.433E\text{-}4\ m^2$$
$$\ell g = 500E\text{-}6\ m$$
$$G = 7.42E\text{-}3\ m$$

$$FF = 1.257$$

raising our low permeability of 50.2 to: 63.1. Not a bad fix.

Power Inductors for Ringing Choke Converters

When we analyzed the RCC back in Chapter 3 the inductance value and current rating of the choke was determined. To actually build up such an experimental circuit one would naturally ask how to construct the inductor so that it functioned properly. Of course an off the shelf inductor from Mouser or Digi-key could be purchased but this would limit our ability to adjust inductance values as we work at the bench.

To build an inductor one of the first questions would be: What size and material core should we use? Naturally the biggest core would work the best but there are practical constraints to consider such as weight, volume and cost. Over the years it became apparent that certain sized ferrite pot cores, when operated at approximately 30kHz in the role of a step-up transformers, could be called on to handle known power levels without overheating. The following list was made from these observations on pot cores. :

Table 7-1: Maximum power VS pot core size

Pot core Size	Maximum Transformer Power level
1408	3 Watts
1811	10
2213	25
2616	50
3019	100
3622	200
4922	400

Although AHV had great success building pot core based high voltage step-up transformers with this rule-of-thumb, there are many other types of cores and materials that can be utilized for the role of an inductor including RM and EC cores, toriodals, iron powder, MMP, or even Sendust cores. The question is what core should we use? If the core is too small the copper losses go up. If the core it too large the costs go up. Is there a way to single out the best core?

Fortunately about 40 years ago, Colonel William McLyman (who we mentioned earlier) working at Jet Propulsion Labs of Cal-Tech did an in-depth study of this question relating to power and core size and we will utilize his techniques here in finding the right sized core to use for both inductors and in the next chapter, transformers.

We shall skip over the idea of making an air core inductor and only look at using a material with a high permeability – this will not only make it smaller and portable but will reduce the EMI as well. All of the cores we will examine concentrate the magnetic field within their structure.

As you probably know there are many cores and materials to choose from, each has their own particular advantage. To help sort out this situation Colonel McLyman defined a term he called, A_P, the a*rea-product.* This was a parameter made from physical dimensions associated with the size of a particular core and the maximum currents it can safely handle. It did NOT involve the material properties other than setting a limit on the maximum B field by composition. Not wanting to plagiarize his work, we will only list some of the important steps he utilized in selecting an appropriate sized core. Those tasked with designing many inductors and transformers should find a copy of his out-of-print book: *Transformer and Inductor Design Handbook* printed in 1988. Treasure this book for the cornucopia of information it presents.

The mathematical definition Colonel McLyman assigned to each core comes from this equation for the area-product:

$$Ap = (\text{Cross-sectional area of the magnetic core})(\text{Winding area}) \tag{7-37}$$

Where we have already worked with the cross-sectional area of the magnetic core, *Ae.* The winding area W_a is a new term and describes the area that the turns will go into.

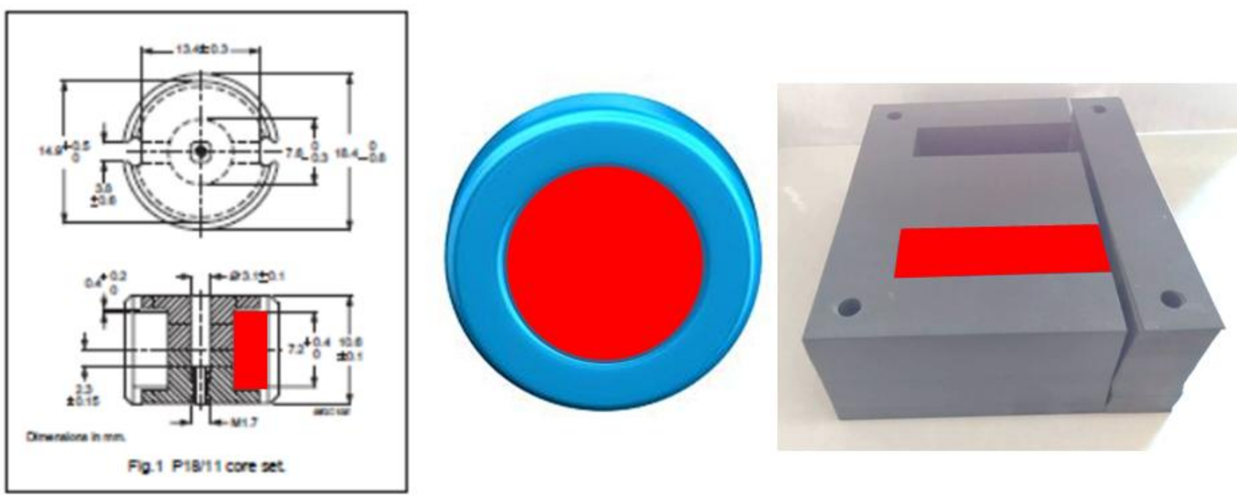


Figure 7.13: Winding area W_a of different cores (in red)

It effectively takes the two important parameters of a magnetic core and merges them together into one so a selection can be made when considering what size to choose. Many core manufacturers such as Magnetics or Ferroxcube list this area-product as a parameter in their catalogs. For example, in Figure 7-14 are tabulated parameters of pot cores a popular type used for high voltage transformers. Notice the term WaAc listed, this is the area- product for the cores listed. The area-product typically has units of cm^4.

		MAGNETIC DATA						HARDWARE
TYPE/SIZE	ORDERING CODE	l_e (mm)	A_e (mm^2)	A_e min (mm^2)	V_e (mm^3)	WaAc (cm^4)	Weight (grams per set)	Bobbins
PC 7/4	**0_40704UG**	9.9	7.0	5.9	69	0.002	0.5	00B0704B1
PC 9/5	**0_40905UG**	12.5	10.1	8.0	126	0.003	0.8	00B090501
PC 11/7	**0_41107UG**	15.5	16.2	13.2	251	0.006	1.8	00B1107B1
PC 11/9	**0_41109UG**	16.2	16.3	13.2	264	0.01	1.9	
PC 14/8	**0_41408UG**	19.8	25.1	19.8	495	0.02	3.2	PCB1408TE
PC 18/11	**0_41811UG**	25.8	43.3	36.0	1,120	0.07	6.4	PCB181111
PC 18/14	**0_41814UG**	29.3	42.6	36.0	1,248	0.09	7.4	
PC 22/13	**0_42213UG**	31.5	63.4	50.9	2,000	0.18	13	PCB221311
PC 26/16	**0_42616UG**	37.6	93.9	77.4	3,530	0.39	20	PCB261611
PC 28/23	**0_42823UG**	48.1	128	101	6,160	0.58	32	00B282301
PC 30/19	**0_43019UG**	45.2	137	116	6,190	0.74	34	PCB301911
PC 34/28	**0_43428UG**	58.1	159	122	9,230	22.4	47	
PC 36/22	**0_43622UG**	53.2	202	172	10,700	1.53	57	PCB362211
PC 42/29	**0_44229UG**	68.6	265	214	18,200	3.68	104	PCB4229L1

Refer to page 62 for additional hardware information.

Figure 7.14: Pot core data

As pot core sizes increase from a 704 (7mm diameter 4mm height) to a 4229 (42 mm diameter to 29mm height), a factor of six in diameter, the area product increases 1840 times, a little more than 6^4. This fourth power relationship corresponds closely with the pot core power listing used by AHV that was found from experience and was mentioned earlier. Other types of ferrite cores such as EC and RM cores also have similar Area-Product listings.

What about the electrical requirements? Our converter has parameters such as frequency and maximum currents that we have to take into consideration to insure the core can handle it. All of this was determined by Colonel McLyman in his inductor equation that gives the *minimum* value of Area-Product that the core should have:

$$Ap = [(2Energy)(1E4) / Bm\ Ku\ Kj)]^{x} \qquad (7\text{-}38)$$

where the *Energy* term is in Joules, the magnetic field *Bm* is the maximum B field that we will operate at (in Tesla), *Ku* is the window utilization number and *Kj* is the current density coefficient. The exponent *x* is different for different cores. His derivation of this equation used parameters that assumed a temperature rise of 25°C for core and copper losses. We will now look at each K factor:

Ku factor: In all applications we acknowledge the fact that no matter how close and tight we wind a bobbin or toroid, there are always air spaces in-between the turns. The *Ku* term takes this into account by looking at several different parameters such as thickness of wire insulation, fill factor, actual window area being utilized and lastly, the use of insulation between layers such as tape or fish paper. Fortunately, there are assumptions that can be made so that *Ku* can be reduced down to a simple number. According to McLyman, a typical value for the copper fraction in the cores window area is:

$$Ku = 0.4$$

When we discuss winding high voltage transformers with small diameter wire such as #46 AWG, this value will drop to 0.3 because the insulation of the wire adds a cross-section that is comparable to the copper area of the conductor. For large lamination cores operating at power line frequencies a *Ku* of 0.4 works very well.

Kj factor: Is a parameter that relates to temperature rise. It will allow us to calculate the current density in the wire we use to fabricate the inductor or transformer. That relationship is given by:

$$J = Kj\, A_P^{\,y} \qquad (7\text{-}39)$$

Values of Kj , x and y (we will use y later) are as follows:

Table 7-2: Parameters used in the Area-Product analysis (from McLyman)

	Bmax	*Kj*	*x*	*y*
Ferrite:	0.25 Tesla	433	1.20	-0.17
Iron Powder:	0.3	403	1.14	-0.12
MMP:	0.3	403	1.14	-0.12
Sendust:	0.4	403	1.14	-0.12
Si-Fe Lamination:	1.2	366	1.14	-0.12
Tape-wound cores:	0.6	250	1.15	-0.13

We will use the area product *Ap* in an analysis of four different inductor materials for a simple ringing choke converter that correspond to examples from previous chapters – just as a design engineer would do and follow the process to completion to determine what core material would be the best to select. Inserting the above values we can generate equations that correspond to the various types of cores listed above. This makes our equations a little easier to use:

Table 7-3: Area product as a function of Energy (from McLyman)

Ferrite pot core:	*Ap*	=	$[461\ Energy]^{1.20}$
Powdered iron toroid:	*Ap*	=	$[413\ Energy]^{1.14}$
MMP toroid:	*Ap*	=	$[413\ Energy]^{1.14}$
Sendust	*Ap*	=	$[310\ Energy]^{1.14}$
Si-Fe Lamination:	*Ap*	=	$[114\ Energy]^{1.14}$
Tape-wound cores:	*Ap*	=	$[333\ Energy]^{1.15}$

The energy is the same no matter what core we use:

$$Energy = ½\ Li^2$$

If L is in Henrys and the current i in Amperes, the Energy term is in Joules. This allows us to determine the size of the core. Let's try an example.

Going back to the Ringing Choke Converter, we can fine tune our list and say that we will either utilize a pot core or a toroid core (because the number of turns is not too great). We will leave the tape-wound and lamination types for other uses due to several reasons. First, lamination transformers are fine for power line frequencies of 60Hz but show excessive eddy current and hysteresis losses at frequencies above 400Hz. Tape-wound cores using Permalloy compositions are more or less limited to frequencies below 20kHz due to eddy current losses. The newer core materials such as amorphous or nanocrystalline types are usable to much higher frequencies, perhaps above 100kHz, but their availability is generally narrow. Therefore, we will limit our search to the first four core materials for the ringing choke converter we explored in Chapter 3.

Example 6: Three watt Ringing Choke Inductor Design:

For one of our examples dealing with ringing choke converters we utilized a 478 uH inductor that had to pass a peak current of 0.679 Amperes when it converted 9 volts to 30 volts at a power level of 3 Watts. This was used to drive an ultraviolet LED array for some sort of medical equipment (replacement of a Wood's Lamp). Operating at 20kHz, the RCC ran with a high efficiency of 91.1%. From the chart of the waveforms shown in Figure 7-6, the current ramps start from zero and rise to 0.679 Amperes peak. In other words, the current has an average level of 0.3395 Amperes with an AC component of +/- 0.3395 Amperes riding ion it. This will come up when we calculate our core losses.

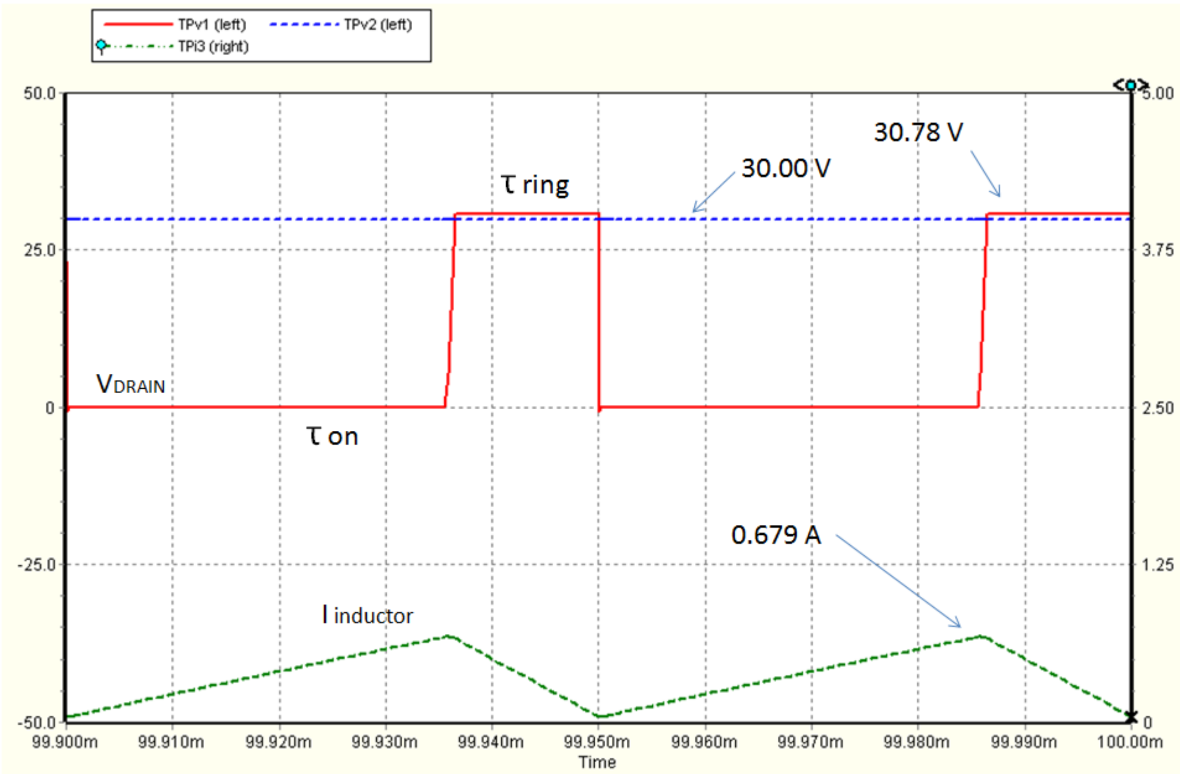


Figure 7.15: 9 to 30V ringing choke converter

Because our L = 478 uH and I peak = 0.679 Amperes we find the Energy term to be:

$$Energy \quad = \quad \tfrac{1}{2} LI^2 \quad = \quad 0.000110 \quad Joules$$

Calculating our area products for the four different cores we are testing as candidates:

Ap	=	$0.0280\ cm^4$	*ferrite pot core*
Ap	=	$0.0295\ cm^4$	*powdered iron*
Ap	=	$0.0295\ cm^4$	*MMP core*
Ap	=	$0.0212\ cm^4$	*Sendust core*

As any engineer would, let's see how each one stacks up in this quest to find the best core to use. The area-product will allow us to select the proper size for each type. Let's examine the four core types.

Ferrite Pot Core:

We need a core with an Ap of: 0.028 cm^4. The 1408 size comes in at 0.02 cm^4, too small and the 1811, the next size larger, has an Ap of 0.07cm^4 quite a bit larger but since there is nothing in-between we will select that core. Here is information concerning the 1811 core:

<u>1811 Core:</u>

Ap	*0.074 cm^4*
Window area:	*0.171 cm^2*
Area of core:	*0.433 cm^2*
Magnetic path length:	*2.58 cm*
Volume:	*1.12 cm^3*
Mass:	*6.0 grams*

Powdered iron:

We need at least an area product of Ap = 0.0295 cm^4. The 050 size powdered iron toroid from `Magnetics has an Ap = 0.042 cm^4, more than we need but this is the smallest available.

<u>Magnetics 050 XFlux toroid:</u>

Ap	*0.042 cm^4*
Window area:	*0.383 cm^2*
Area of core:	*0.109 cm^2*
Magnetic path length:	*3.12 cm*
Volume:	*0.34 cm^3*
Mass:	*2.3 grams*

MMP core:

We need an Ap of at least 0.0295 cm^4. The 050 MMP core from Magnetics with an Ap of 0.042 cm^4 will work.

<u>Magnetics 050 MMP core parameters:</u>

Ap	*0.042 cm^4*
Window area:	*0.383 cm^2*
Area of core:	*0.109 cm^2*
Magnetic path length:	*3.12 cm*
Volume:	*0.34 cm^3*
Mass:	*2.5 grams*

Sendust:

We need Ap = 0.0212 cm^4, a little bit lower than the others because we are running it at a higher magnetic field, the 040 Kool Mu (Sendust) core from Magnetics will work, it has an Ap value of 0.024 cm^4 .

Magnetics 040 Kool Mu (Sendust) Core parameters:

Ap	*0.024* cm^4
Window area:	*0.268* cm^2
Area of core:	*0.090* cm^2
Magnetic path length:	*2.70 cm*
Volume:	*0.243* cm^3
Mass:	*1.53 grams*

Now that we have our four candidate cores selected we need to determine the number of turns for each core to achieve the required inductance of 478 µH to make our step-up converter. This will require us to look at the maximum *current density* which according to McLyman is given by the following equation that allows a copper wire temperature rise of 25 °C, repeating the equation for current density:

$$J = Kj\ Ap^{\,y} \qquad (7\text{-}40)$$

We must insert the core data of the core we will utilize where we use K_J from Table 7-2. Insert what corresponds to the core you have selected.

Ferrite pot core (1811 core size):	$J = (433)(0.07)^{-0.17}$	=	*680 Amperes/*cm^2
Powdered iron (050 core size):	$J = (403)(0.042)^{-0.12}$	=	*589 Amperes/*cm^2
MMP core (050 core size):	$J = (403)(0.042)^{-0.12}$	=	*589 Amperes/*cm^2
Sendust (040 core size):	$J = (403)(0.024)^{-0.12}$	=	*630 Amperes/*cm^2

This will limit the temperature rise in the copper to less than 25°C. From our simulation (in Chapter 3) we know the maximum current of 0.679 Amperes will flow allowing us to calculate the bare wire size needed:

$$Awire_{(Bare)} = Imax / J \qquad (7\text{-}41)$$

Calculating out the bare cross-sectional area needed gives us the wire size (when looking at the wire table in Appendix A in back of this chapter).

Ferrite pot core: $Awire_{(Bare)} = (0.679A)/(680A/cm^2) = 0.990E\text{-}3cm^2 \Rightarrow$ *#27 AWG*
Powdered iron: $Awire_{(Bare)} = (0.679A)/(589A/cm^2) = 1.153E\text{-}3cm^2 \Rightarrow$ *#26 AWG*
MMP core: $Awire_{(Bare)} = (0.679A)/(589A/cm^2) = 1.153E\text{-}3cm^2 \Rightarrow$ *#26 AWG*
Sendust core: $Awire_{(Bare)} = (0.679A)/(630A/cm^2) = 1.078E\text{-}3cm^2 \Rightarrow$ *#26 AWG*

Our selection was based on the following information:

Information:	*#27 AWG bare copper cross sectional area of:*	$1.021E\text{-}3\ cm^2$.
	#26 AWG bare copper cross sectional area of:	$1.280E\text{-}3\ cm^2$
and:	*#27 AWG insulated wire cross sectional area of:*	$1.313E\text{-}3\ cm^2$.
	#26 AWG insulated wire cross sectional area of:	$1.603E\text{-}3\ cm^2$

Now that we know the wire size we can find approximately how many turns of wire we need on the selected core. We need to use the insulated wire area values because that is the closest to real-world and as you can see, the insulation does take up some of the volume. Remember, we are working four different core materials at the same time to see which one will work the best.

Using this data we can determine the maximum number of turns we can place on each selected core:

$$N = Wa\, S_2 / Awire \qquad (7\text{-}42)$$

where S_2 is a fill factor of 0.6 that takes into account the tightness of the winding.

Ferrite pot core:	*N*	=	*(0.171)(0.6)/(0.00102)*	=	*78 turns #27 AWG*
Powdered iron:	*N*	=	*(0.383)(0.6)/(0.00128)*	=	*143 turns #26 AWG*
MMP core:	*N*	=	*(0.383)(0.6)/(0.00128)*	=	*143 turns #26 AWG*
Sendust:	*N*	=	*(0.268)(0.6)/(0.00128)*	=	*100 turns #26 AWG*

Now that we know the *approximate* number of turns that will fit in the winding area of our core and generate a temperature rise less than 25 °C, the required approximate A_L value can be determined. We say approximate because at this point the exact formulation one can buy for the toroid is set in certain steps by the manufacturer. Unlike pot cores the three different toroidal powder cores cannot be adjusted for A_L values, they must be purchased in set increments. We will pick the one that allows us to create the inductor we need. Using our inductance equation:

$$A_L = L / N^2$$

we find that our cores must have something close to the following A_L values:

Ferrite pot core:	A_L	=	$(478\ \mu H) / 78^2$	=	$78\ nH / Turn^2$
Powdered iron:	A_L	=	$(478\ \mu H) / 143^2$	=	$23\ nH / Turn^2$
MMP core:	A_L	=	$(478\ \mu H) / 143^2$	=	$23\ nH / Turn^2$
Sendust:	A_L	=	$(478\ \mu H) / 100^2$	=	$48\ nH / Turn^2$

Because toroidal cores are only made in certain A_L values we will have to select a core from the catalog that has the closest value to what we need. With a ferrite pot core we can adjust the A_L value by simply inserting a gap in-between the core halves.

Gapping the ferrite core:
As mentioned earlier, you can purchase pre-gapped ferrite cores where the center leg is shorter. This presents a gap in the middle when the two core halves are placed together but these gaps only come in about a half dozen set steps from “no gap” to 710 µm for the 1811 core size.

Determining the total gap:
There is a very good method to determine the gap. We will utilize the gapped data and plot the A_L values as a function of gap as we did in Figure 7-9. From this a TRENDLINE can be made and the correct gap determined. For example, the TRENDLINE equation for A_L values of the 1811 core with 3C81 material gave (from chart 7-9):

$$A_L = 26447\ (\text{gap in } \mu m)^{-0.851} \qquad A_L \text{ in } nH/T^2$$

Because we are looking to determine the gap for an A_L of 78 nH/T^2 we have, using logs:

$$\begin{aligned}
A_L &= 26447\ (\text{gap in } \mu m)^{-0.851} \\
\log_{10}(A_L) &= -0.851 \log_{10}(\text{gap in } \mu m) + \log_{10}(26447) \\
\log_{10}(78) &= -0.851 \log_{10}(\text{gap in } \mu m) + 4.422 \\
1.892 &= -0.851 \log_{10}(\text{gap in } \mu m) + 4.422 \\
-2.53 &= -0.851 \log_{10}(\text{gap in } \mu m) \\
2.97 &= \log_{10}(\text{gap in } \mu m) \\
\text{total gap in } \mu m &= 10^{2.97} \\
\text{total gap in } \mu m &= 939\ \mu m \ \ (0.037 \text{ inches})
\end{aligned}$$

We could fabricate our inductor out of two un-gapped cores by using a 0.0185 inch gapping material between the core halves. Because this may be a tall order to get such a specific gap, it is probably easier to use a 0.02 inch gap material which will give us a total gap of 1016 µm. Let’s set this in stone and determine both A_L and $\mu_{e\ gapped}$ for this gap (1016 µm) from the TRENDLINES:

$$A_L = 26447\ (\text{gap in } \mu m)^{-0.851}$$
$$A_L = 73\ nH/T^2$$

Our gapped effective permeability is:

$$\mu_{e\ gapped} = 12883\ (\text{gap in } \mu m)^{-0.856}$$
$$\mu_{e\ gapped} = 34.4$$

Knowing our AL value, we can determine the number of turns we will need:

$$L = A_L N^2$$
$$478\ \mu H = (73E\text{-}9)(N^2)$$
$$N = 81 \text{ turns \#27 AWG one section bobbin.}$$

Let's test it using the "short" inductance equation with $\mu_{e\ gapped}$:

$$L = \mu_{e\ gapped}\ \mu o\ N^2\ (Ae) / \ell e$$
$$L = (34.4)(4\pi E\text{-}7)(81^2)(0.433E\text{-}4) / (0.0258)$$
$$L = 476\ \mu H$$

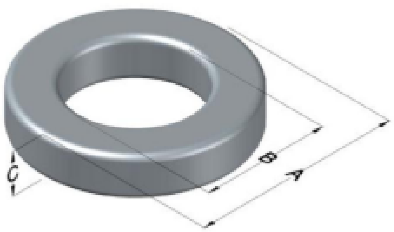
If we select the ferrite core, it looks like it will be an un-gapped 1811 pot core 3C81 material with a 1016 µm total gap made from a piece of 20 mil insulation material for use on the center post having 81 turns of #27 AWG copper wire.

Powdered Iron cores

MAGNETICS®

0078051A7

110 Delta Drive
Pittsburgh, PA 15238
NAFTA Sales: (1)800-245-3984
HK Sales : (852)3102-9337
magnetics@spang.com
www.mag-inc.com



XFLUX Permeability (µ)	A_L (nH/T²)	Core Marking			Coating Color
		Lot Number	Part Number	Inductance Grade	
60	27 ± 8%	XXXXXX	051A7	N/A	Brown

Dimensions	Uncoated		Coated Limits			Packaging
	(mm)	(in)	(mm)	(in)		
OD (A)	12.70	0.500	13.46	0.530	max	Bulk Pack 4 bags/box Box Qty= 5000 pcs
ID (B)	7.62	0.300	6.99	0.275	min	
HT (C)	4.75	0.187	5.51	0.217	max	

Electrical Characteristics			Physical Characteristics						
Watt Loss @ 50 kHz, 100mT max(mW/cm³)	DC Bias min (oersteds)		Voltage Breakdown wire to wire min (VAC)	Break Strength min (kg)	Window Area WA(mm²)	Cross Section Ae (mm²)	Path Length Le (mm)	Volume Ve (mm³)	Weight (g)
	80%	50%							
725	80.0	150	1250	15.0	38.3	10.9	31.2	340	2.5000

Winding Information							Temperature Rating
Winding Length Per Turn				Wound Coil Dimensions (mm)			
Winding Factor	(mm)	Winding Factor	(mm)	40% Winding Factor	OD	14.6	Curie Temp: 700˚C
					HT	7.66	Coating Temp (Continuous up to): 200˚C
0%	17.5	40%	21.1	Completely Full Window	Max OD	18.2	Notes:
20%	19.3	45%	21.7		Max HT	11.5	
25%	19.8	50%	22.1	Surface Area (mm²)			

Figure 7-16: 050 sized iron powder core

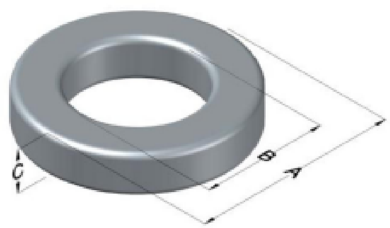
For the powdered iron core we need an A_L vlaue of 23 nH/T^2. The magnetics core 078051A7 with an effective permeability of μ = 60 has an A_L value of 27 nH/T^2, which is close. We can calculate the exact number of turns that will give us 478 uH. The relative permeability of this core is set by its composition to be sixty and therefore we cannot adjust it as we did with the ferrite pot core.

$$L = A_L N^2$$
$$478\ \mu H = (27E\text{-}9)(N^2)$$
$$N = 133 \text{ turns \#26 AWG}$$

MPP cores:

C055051A2

110 Delta Drive
Pittsburgh, PA 15238
NAFTA Sales: (1)800-245-3984
HK Sales : (852)3102-9337
magnetics@spang.com
www.mag-inc.com



MPP Permeability (µ)	A_L (nH/T^2)	Core Marking			Coating Color
		Lot Number	Part Number	Inductance Grade	
60	27 ± 8%	XXXXXX	051A2	X	Gray

Dimensions	Uncoated		Coated Limits			Packaging
	(mm)	(in)	(mm)	(in)		
OD (A)	12.70	0.500	13.46	0.530	max	Bulk Pack 4 bags/box Box Qty= 5000 pcs
ID (B)	7.62	0.300	6.99	0.275	min	
HT (C)	4.75	0.187	5.51	0.217	max	

Electrical Characteristics			Physical Characteristics							
Watt Loss @ 100 kHz, 100mT max(mW/cm^3)	DC Bias min (oersteds)		Voltage Breakdown wire to wire min (V_{AC})	Break Strength min (kg)	Window Area W_A(mm^2)	Cross Section A_e (mm^2)	Path Length L_e (mm)	Volume V_e (mm^3)	Weight (g)
	80%	50%							
700	50.0	94.0	1250	15.0	38.3	10.9	31.2	340	2.9140

Winding Information							Temperature Rating
Winding Length Per Turn				Wound Coil Dimensions (mm)			Curie Temp: 460 ˚C
Winding Factor	(mm)	Winding Factor	(mm)	40% Winding Factor	OD	14.6	Coating Temp (Continuous up to): 200 ˚C
					HT	7.66	
0%	17.5	40%	21.1	Completely Full Window	Max OD	18.2	Notes:
20%	19.3	45%	21.7		Max HT	11.5	
25%	19.8	50%	22.1	Surface Area (mm^2)			

Figure 7-17: MPP core

For the MPP core we need an A_L vlaue of 23 nH/T^2. The Magnetics core C055051A2 has an A_L value of 27 nH/T^2, which is close. Again, it is a core with an effective permeability of 60. We can calculate the exact number of turns that will give us 478 uH. The relative permeability of this core is set by its composition to be µ=60.

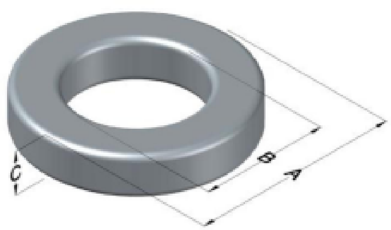
$$L = A_L N^2$$
$$478\ \mu H = (27E\text{-}9)(N^2)$$
$$N = 133 \text{ turns \#26 AWG}$$

Sendust cores:

MAGNETICS®

0077130A7

110 Delta Drive
Pittsburgh, PA 15238
NAFTA Sales: (1)800-245-3984
HK Sales : (852)3102-9337
magnetics@spang.com
www.mag-inc.com



Kool Mμ Permeability (μ)	A_L (nH/T²)	Core Marking			Coating Color
		Lot Number	Part Number	Inductance Grade	
125	53 ± 12%	XXXXXX	130A7	N/A	Black

Dimensions	Uncoated		Coated Limits			Packaging
	(mm)	(in)	(mm)	(in)		
OD (A)	11.18	0.440	11.81	0.465	max	Bulk Pack 4 bags/box Box Qty= 6000 pcs
ID (B)	6.35	0.250	5.84	0.230	min	
HT (C)	3.96	0.156	4.60	0.181	max	

Electrical Characteristics			Physical Characteristics						
Watt Loss @ 100 kHz, 100mT max(mW/cm³)	DC Bias min (oersteds)		Voltage Breakdown wire to wire min (V_{AC})	Break Strength min (kg)	Window Area W_A(mm²)	Cross Section A_e (mm²)	Path Length L_e (mm)	Volume V_e (mm³)	Weight (g)
	80%	50%							
750	14.0	35.0	1250	12.0	26.8	9.06	26.9	244	1.5000

Winding Information							Temperature Rating
Winding Length Per Turn				Wound Coil Dimensions (mm)			Curie Temp: 500˚C
Winding Factor	(mm)	Winding Factor	(mm)	40% Winding Factor	OD	12.9	Coating Temp (Continuous up to): 200˚C
					HT	6.53	
				Completely Full Window	Max OD	15.7	Notes:
0%	15.2	40%	18.1		Max HT	8.97	
20%	16.7	45%	18.6	Surface Area (mm²)			
25%	17.0	50%	19.0				

Figure 7-18: Sendust core

For the Sendust core we need an A_L vlaue of 48 nH/T^2. The Magnetics core 0077130A7 has an A_L value of 53 nH/T^2, which is close. This core has an effective permeability of 125. We can calculate the exact number of turns that will give us 478 uH. The relative permeability of this core is set by its composition to be μ=125 and like the previous toroids we cannot change it.

$$L = A_L N^2$$

$$478\ \mu H = (48E\text{-}9)(N^2)$$

$$N = 100 \text{ turns \#26 AWG}$$

Copper Losses

The McLyman analysis we are using limits the copper temperature rise to 25 degrees C. Nevertheless, we should calculate the copper wire loss for our four different cores to see how they compare. All we have to do is determine the DC resistance for each winding and use:

$$Power = i^2 R$$

To determine the loss we will use the maximum current value in our calculation of 0.679 Amperes. While this is not exactly correct (because the current level is usually much below this number and even goes to zero at one point) we will err on the side of caution.

Table 7-4: Copper losses for each core type

	Core	Turns	AWG	MLT (cm/T)	Wire length (cm)	Ω/cm (μ – Ω)	R (Ohms)	loss (W)
Ferrite pot core:	1811	81	27	3.66	296cm	1687	0.50Ohms	0.23
Powdered iron:	050	133	26	2.11	281cm	1345	0.38Ohms	0.18
MPP core:	050	133	26	2.11	281cm	1345	0.38Ohms	0.18
Sendust:	040	100	26	1.81	181cm	1345	0.24Ohms	0.11

showing that our ferrite core displays the most heating and the sendust core the least. Now to calculate the core losses.

Core losses are now calculated: (f = 20kHz)

Below is a profile of the inductor current in our RCC for Example 1 (from Chapter 3):

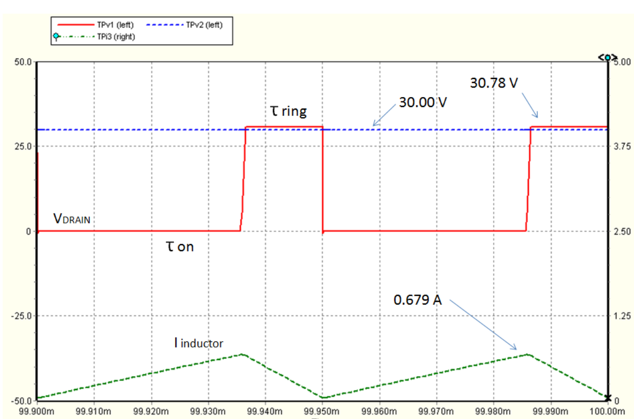


Figure 7-19: Inductor current (green)

Because the current swings from zero to 0.679 Amperes, it has a “DC component” of 0.3395 Amperes and the AC component of +/- 0.3395 Amperes.

Core loss is generated only by the AC component. A constant DC flux can only cause Ohmic losses which are not related to the core and we have calculated these already. Magnetic materials by their very nature have both eddy current and hysteresis losses caused by changing B fields that sap energy from the driving circuitry and wind up as heat. Core loss density is a function of one-half of the AC flux swing ($1/2\ \Delta B = B_{peak}$)

1811 Ferrite pot core loss:
Because we are driving our inductor with an AC waveform (see Figure 7.19), the B field created can be determine from Faraday’s law:

$$Bmax = Vp / NAe2\pi f$$

we can easily calculate the *Bmax* swing. Plugging in the numbers for the 1811 pot core and data from Figure 7.19 we have:

$$Bmax = (30)/((81(4.33E-5)(2\pi)(20{,}000)$$
$$Bmax = 0.06\ Tesla)$$

Looking at the core loss chart (from Ferroxcube) for the 3C81 material, although a frequency of 20kHz is not listed, a region close by (red dot) may be assumed:

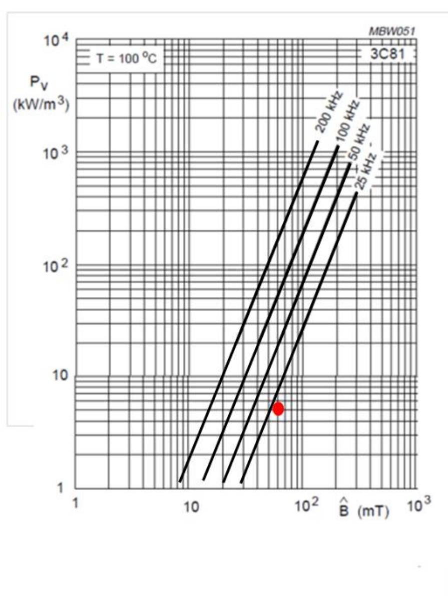


Figure 7.20: Loss chart 3C81

And obtain a loss of about 5mW/cm^3 .   Now, using the volume of our 1811 core we can determine the core losses:

	$B_{AC\ peak}$	loss (20kHz)	volume	loss
	-------	---------------	--------------	--------
Ferrite pot core:	0.06 Tesla	5 mW/cm^3	1.12 cm^3	0.0056 W

Powdered Iron core loss:

For our powdered iron core, the 0078051A7 we again can use Faraday's equation to find the B field:

$$Bmax = Vp / NAe 2\pi f$$
$$Bmax = (30)/((133)(1.09E\text{-}5)(2\pi)(20{,}000)$$
$$Bmax = 0.164\ Tesla)$$

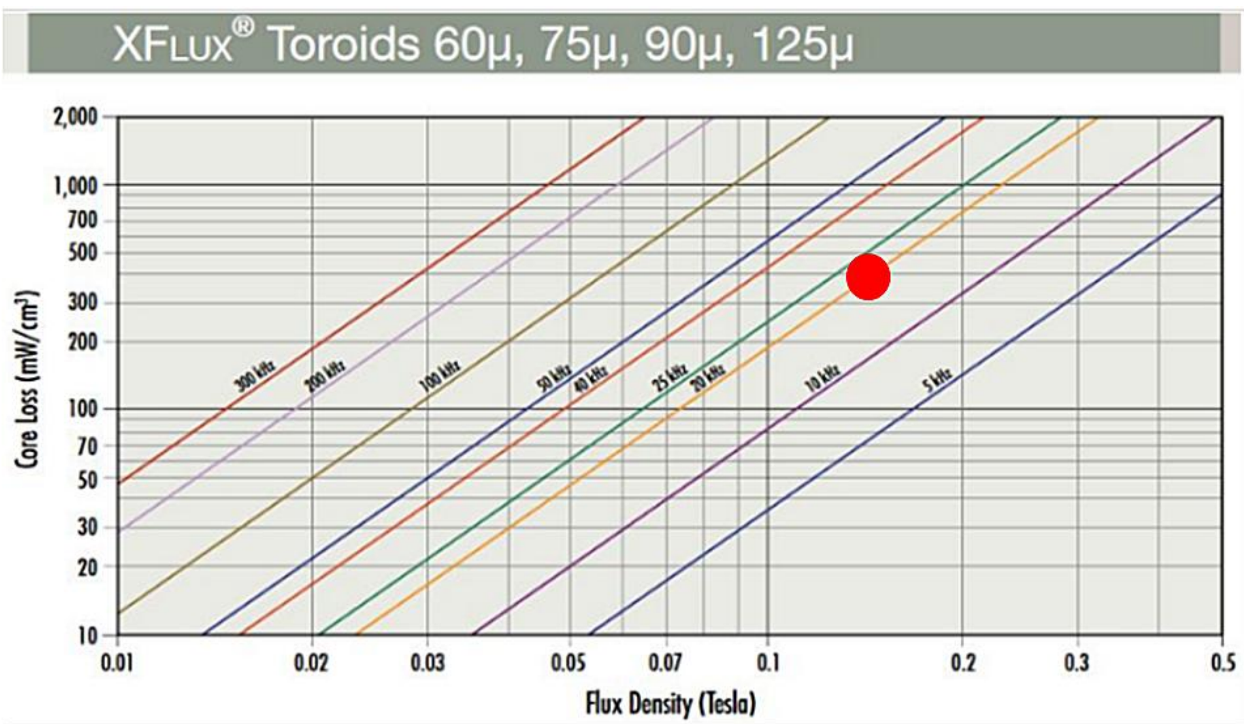


Figure 7.21: Powdered iron loss chart

	B_{ACpeak}	loss (from chart)	volume	loss
	----------	-------------------	-----------	--------
Iron powder core:	0.164 Tesla	400 mW/cm^3	0.34 cm^3	0.136 W

MPP core loss:

For our MPP core, the C0055051A2 we again can use Faraday's equation to find the B field:

$$Bmax = Vp / NAe 2\pi f$$
$$Bmax = (30)/((133)(1.09E\text{-}5)(2\pi)(20{,}000)$$
$$Bmax = 0.164 \text{ Tesla})$$

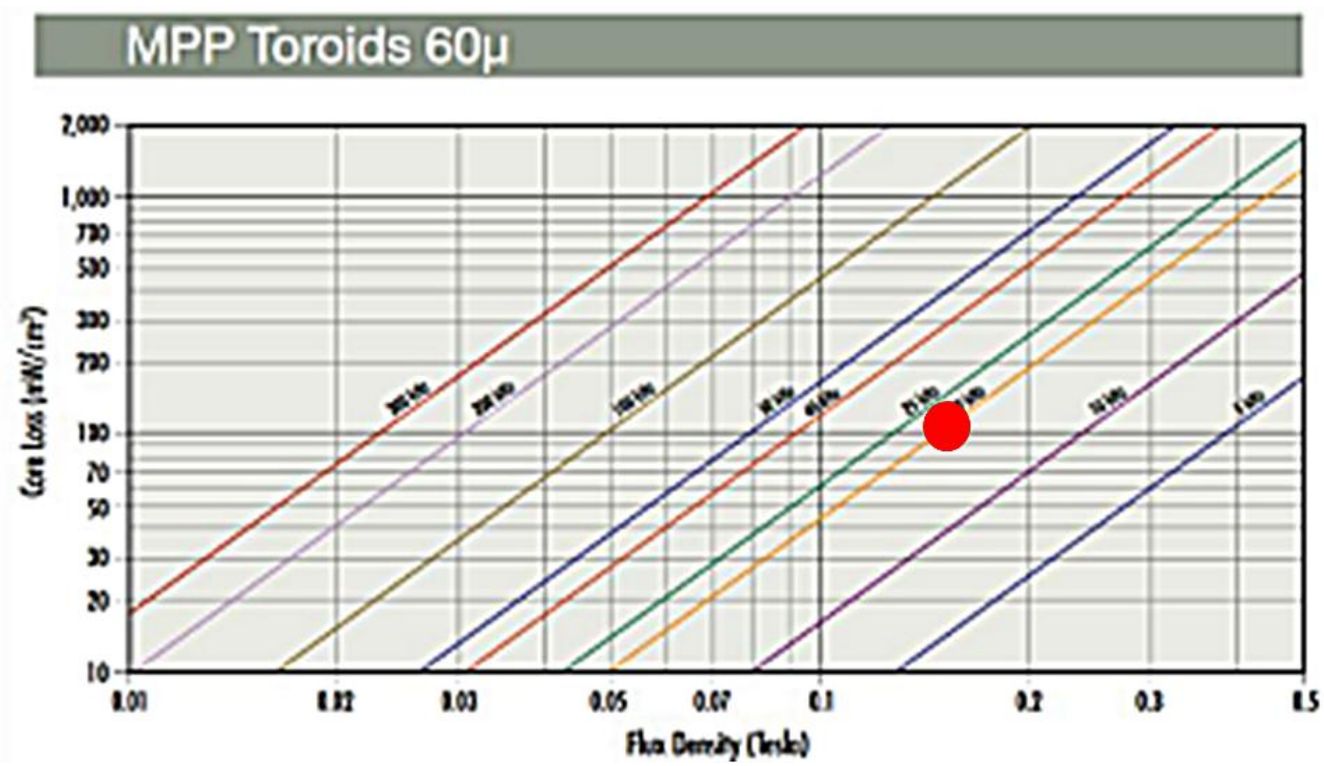


Figure 7.22: MPP Core loss chart

	B_{ACpeak}	loss (from chart)	volume	loss
	----------	-------------------	-----------	--------
MPP core :	0.164 Tesla	110 mW/cm^3	0.34 cm^3	0.037 W

Sendust core loss:

For our Sendust core, the 0077130A7 we again can use Faraday's equation to find the B field:

$$Bmax = Vp /NAe2\pi f$$
$$Bmax = (30)/((100)(0.9E\text{-}5)(2\pi)(20{,}000)$$
$$Bmax = 0.265 \text{ Tesla})$$

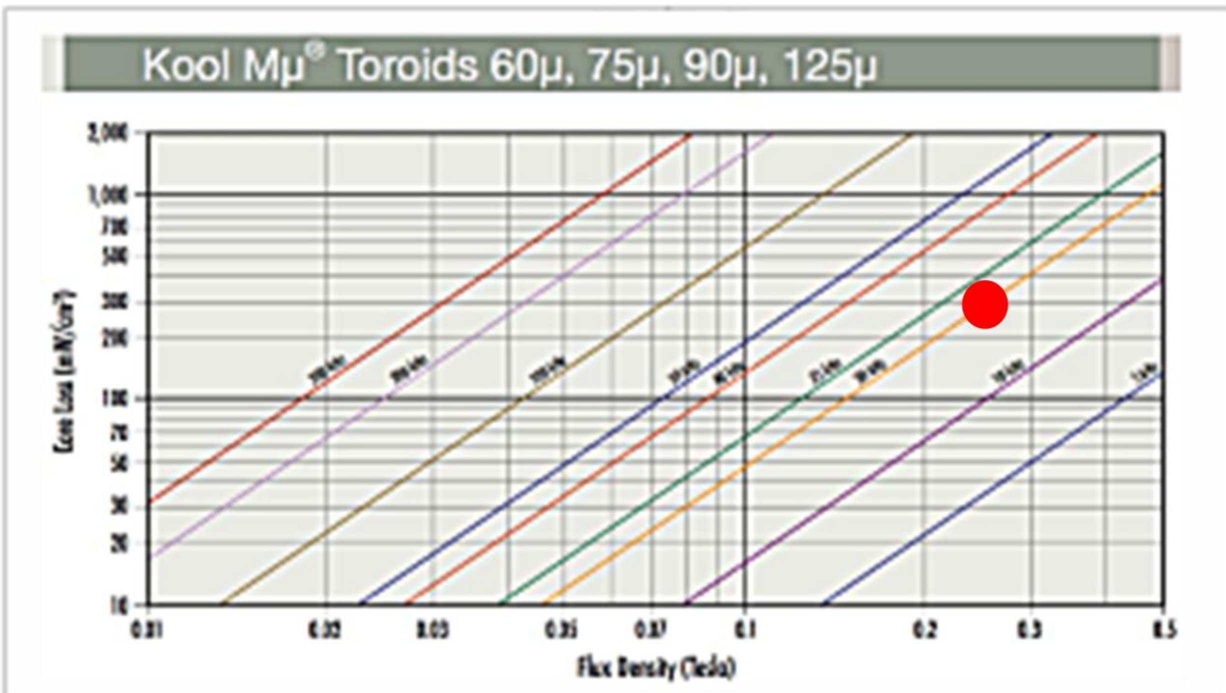


Figure 7.23: Sendust core loss chart

	B_{ACpeak}	loss (from chart)	volume	loss
	-------	---------------	--------------	--------
Sendust core loss:	0.265 Tesla	300 mW/cm^3	0.243 cm^3	0.073 W

Table 7-5: Summation of losses for our 478 uH inductor:

		Copper loss	Core loss	Sum of loss
1811	Ferrite pot core:	0.23 W	0.0056 Watts	0.24 Watts
0078051A7	Powdered iron:	0.18 W	0.136	0.32
0055051A2	MMP core:	0.18 W	0.037	0.22
0077130A7	Sendust:	0.11W	0.073	0.18

This analysis shows that the Sendust core will offer the lowest energy loss.

FROM MAGNETICS ONLINE:
An online calculator furnished by Magnetics allows us to compare our direct calculations with theirs obtained by a proprietary analysis technique:

Table 7-6: Core Losses from the online calculator

		Copper loss	Core loss	Sum of loss
1811	Ferrite pot core:	0.24 W	0.0042 Watts	0.24 Watts
78054	Powdered iron:	0.18 W	0.085	0.27
55045	MMP core:	0.18 W	0.027	0.21
77050	Sendust:	0.11W	0.04	0.15

According to Magnetics, it looks like the Sendust core will dissipate the least amount of heat confirming our earlier loss analysis.

From this analysis, the basic inductor core costs are:

Ferrite pot core: Total including bobbin: $2.99

Powdered iron: Eaton: CTX500-1-52LP-R 500uH $5.93

MPP core: Cores: MMP: API Delevan Inc.PTHF500-894 $4.41
from Micro-Semiconductor

Sendust: Bourns: 2100LL-471-H-RC 470 uH 2.3A $ 3.40

How much will the inductor change when current is flowing through?
All of the above may be invalidated if we find out that our inductance varies by an unusable amount. All inductors made with ferromagnetic material have permeabilities that change over temperature, field strength and time. As the core circles around it's BH curve the effective mu is constantly changing.

Ferrite 3C81 inductor change:
In order to see what the change in inductance is we must see how the permeability changes with B field. Figure 7.21 shows the BH excitation based on a B field maximum of 0.06 Tesla.

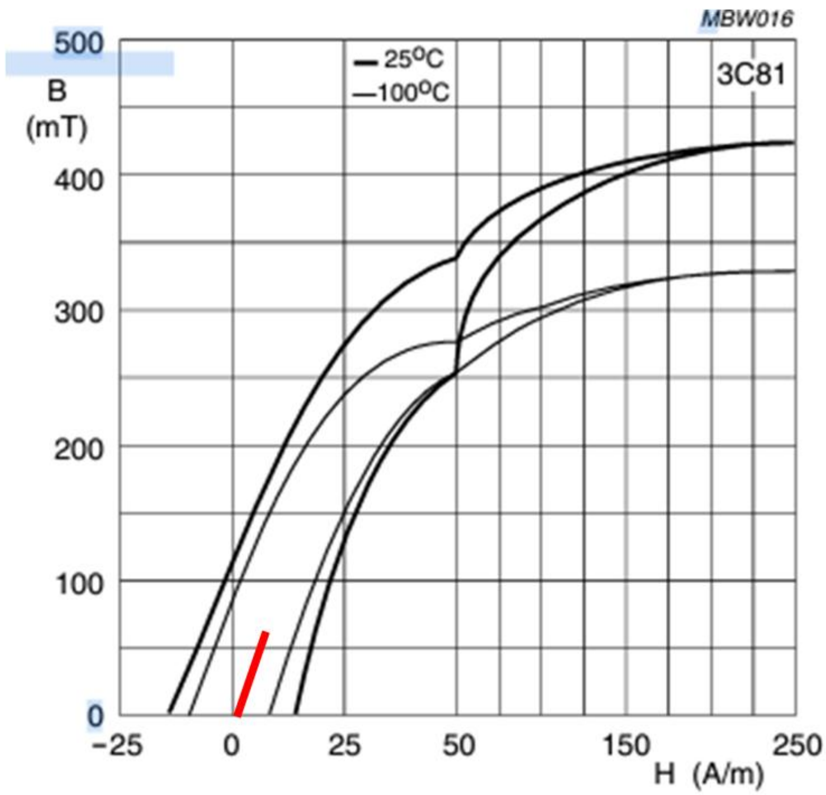


Figure 7.24: Maximum BH curve for 3 Watt converter

The B field excitation is in one direction only in the core because the current flow is uni-directional. Because the B field maximum is far away from any Bsat values (B > 0.25 T) we can rest assured that the inductance calculated of 478 μH will remain very close to that value.

Iron Core Inductance Drop-Off:

The iron powder core, 78051A7, shows the following roll-off in inductance with a calculation of the value of current-turns as presented on the Magnetics website for the core we are using. Here the x axis is not H field but simply Ampere-Turns, and if we insert the values of (0.679)(133) = 90.3 it looks like we will not suffer any drop in AL value more than a few percent. Remember we had used 27 nH/T^2 in our calculation for the number of turns required.

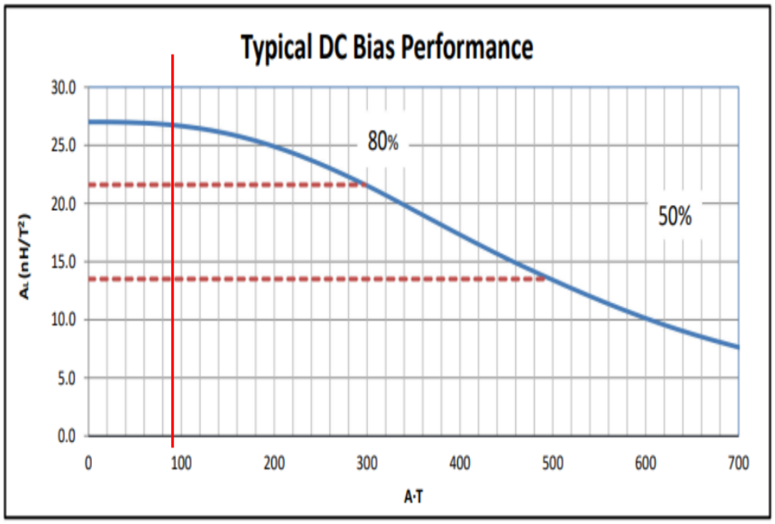


Figure 7.25: AL drop off as a function of current in the 78051A7 iron powder core

MPP Core Inductance Drop-Off:

The molypermalloy powder (MPP) core (MPP) powder core, the C0055051A2 shows the following roll-off in inductance. It looks like we will suffer a drop in AL value from 27 to 25 nH/T^2 and may want to take this into consideration when calculating the final number of turns.

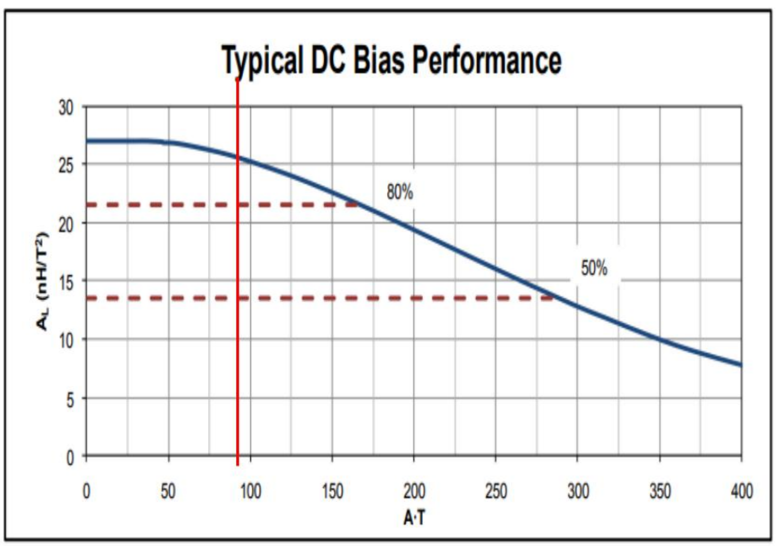


Figure 7.26: AL drop off as a function of current in the C0055051A2 MPP core

Sendust Core Inductance Change

The Sendust core ,0077130A7, shows the following roll-off in inductance with DC component of waveform. If we insert the values of (0.679)(100) = 67.9 on the x axis, it looks like our A_L value will drop from 53 to 33. This can be adjusted for by increasing the turns from 100 to 126 – which further decreases the A_L value.

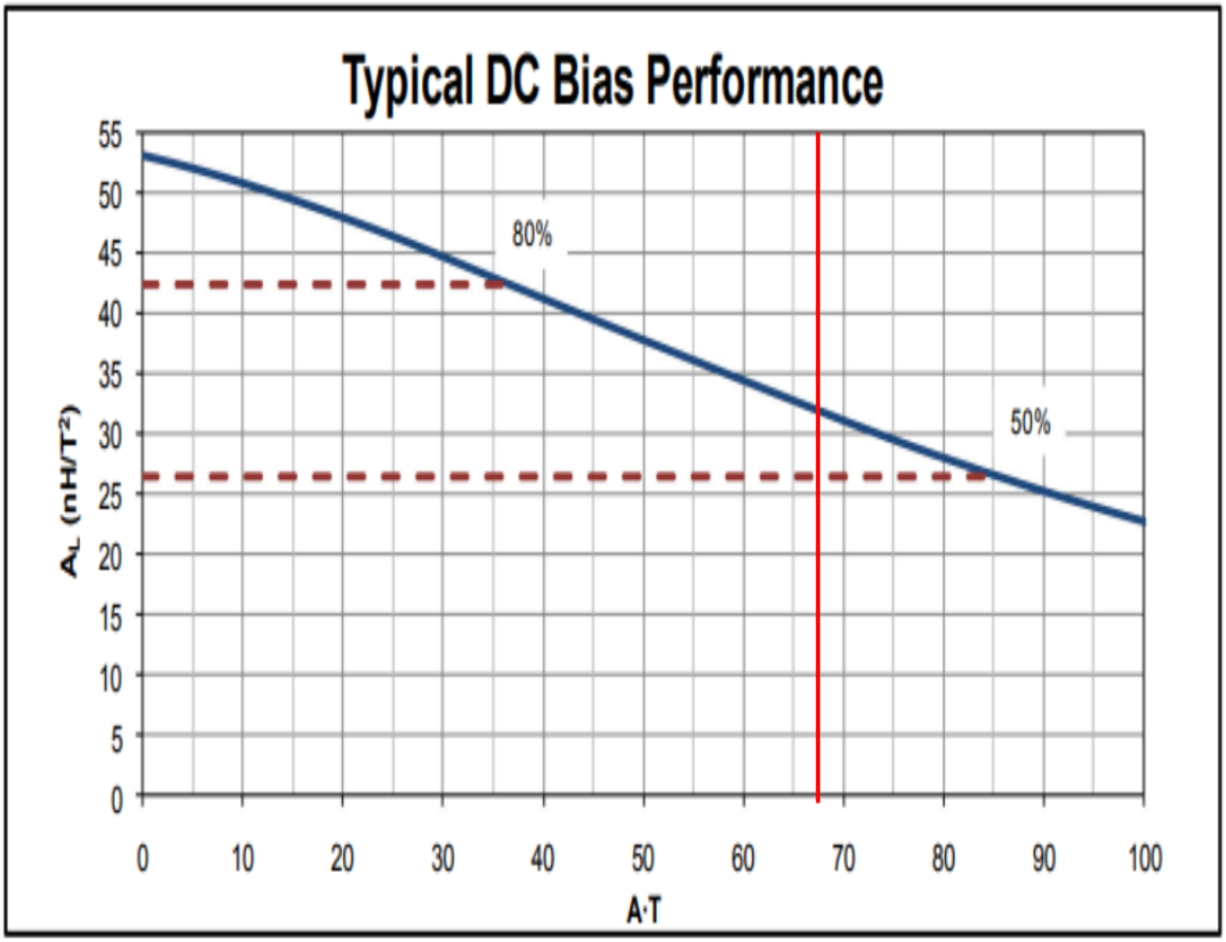


Figure 7.27: Sendust core inductance drop-off

Analysis and Final Core Selection for the 3 Watt converter

The fact that the Sendust core drops 62% in its A_L value sets off a danger signal when selecting this core. Even though it offers the lowest losses at 0.15 Watts, the runner up in that area is the MPP core with only a slight more loss at 0.22 Watts. The good feature of the MPP core is that it doesn't change more than a few percent and during an overload condition can expect to hold the inductance more than the Sendust core. This may be a safety factor. If cost is not a gating item the MPP core would be selected.

Input Line Chokes

To meet certain EMI specifications a power converter operated either from an AC power main or a DC bus must limit the amount of electrical noise placed back onto its input line. This was not so much of a problem when 60Hz power transformers were utilized to convert 120 VAC to the needed voltage. With the advent of switching power supplies in the 1980's, elaborate filtering was necessary to prevent high frequency signals from traveling outwards from the power stage making the power line into an antenna and radiating the energy further onward. Not only can this unwanted EMI noise affect communications but sensitive instruments such as medical devices as well. For example, a sensitive 16-bit analog to digital converter used in a signal path of, say, a medical system should not pick up switching noise coming from the supply that provides power. Imagine an EKG machine used in proximity to a noisy 20kHz switching power supply – the important signals from the heart may be overshadowed by the noisy switching power supply. In addition, the problem is severe with airborne equipment, even to this day, certain types of electronics is forbidden for use on commercial flights. Military airborne systems must meet specified sections on conducted emissions of MIL-E-461.

The manufacturer of any power converters have to fulfill certain EMI regulations that are put in place by governments all over the world to protect reliable functionality of different electronic systems simultaneously.

EMI comes in two forms: radiated EMI and conducted EMI. The most effective ways of reducing radiated EMI is to optimize the PCB layout and to use a shielded metal box to house the converter in. However, this may not be practical and in most cases is very costly.

Conducted EMI is typically attenuated by additional input line filtering. The reduction of noisy switching signals is a black magic art. It is almost impossible to simulate because computer models are not available for every configuration of converter placed inside of a box so big. When power switching devices operate on and off, they will generate large sometimes discontinuous currents. These currents will appear at the input of a buck converter, the output of a boost converter, and at both the input and output of flyback and buck-boost converter designs.

About the only sure fire way of minimizing the output of conducted emissions back onto the input line is to place as large an inductor in series with the power feed lines. A pi-filter arrangement will help quench the noise and should be designed in.

Usually the problem is a matter of space allocation. The input filter is usually left to the last to allocate room for, where the converter topology is always given first preference. With this in mind, the following lists the effective impedance of inductors as a function of frequency:

Table 7-7: Inductor impedance values

Frequency	Inductance	Impedance
20kHz	100 µH	12.5 Ohm
200 kHz	100 µH	125 Ohms
20kHz	1 mH	125 Ohms
200 kHz	1 mH	1.25K Ohms
20kHz	10 mH	1.25K Ohm
200 kHz	10 mH	12.5K Ohms

Considering an 10 Watt AC converter operating from 120 VDC, the load impedance as seen by the power plug is:

$$\textit{Load resistance:} \quad 120^2/10 \quad = \quad 1.4K\ Ohms$$

You may make the assumption that the signals start with a generator of this impedance. If we put an inductor of 100 µH in series with the lines, even without the capacitance, the EMI will nearly drop in half if the converter is operating at 20kHz. A 1 mH inductor will reduce the EMI by a factor of ten. Adding capacitors from line to line will certainly help as well.

Example 7:

Design of 10 mH INPUT LINE INDUCTOR for a 10 Watt AC line product

A 10 Watt converter operating from the 120 VAC line will draw in approximately 100 mA. If there is digital circuitry inside the power converter the AC line must be stepped down to low voltage to operate the semiconductors. This usually means some type of switching converter. Because Wire has a self inductance of about 400 nH / foot we will take this into consideration.

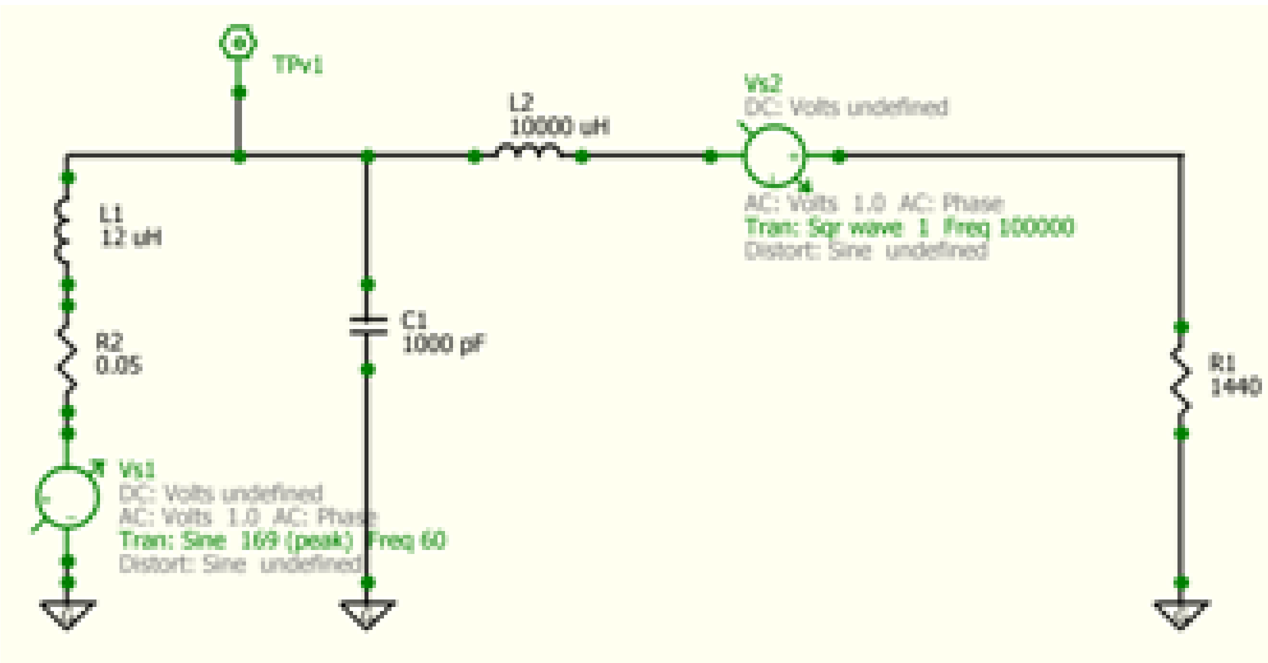


Figure 7.28: SPICE test circuit for EMI reduction

Figure 7-46 shows the simple SPICE test circuit one can use to determine the effect of filter values on conducted EMI noise. Here we have a 60Hz power source operating into a 10 Watt resistive load. Both the line cord resistance and inductance is assumed to be 0.05 Ohms and 12 uH respectively (1440 Ohms). The source of the EMI noise is a square wave of 1.0 Vpp shown at Vs2. C1 and L2 are the filter we are attempting to design. Therer is a practicle value to C1 where capacitors above 0.01 uF tend to draw currents that are in the same order of magnitude as the load for a 10 Watt device (although they are 90 ° out of phase with the voltage).

Capacitance	Current drawn
0.001	45 uA
0.01	450 uA
0.1	4.5 mA
1.0	45 mA

and will cause Ohmic losses when operating on any resistance in the path. Figure 7-47 shows the effect of place various inductor values in series with the line on the unit converter side of the capacitor.

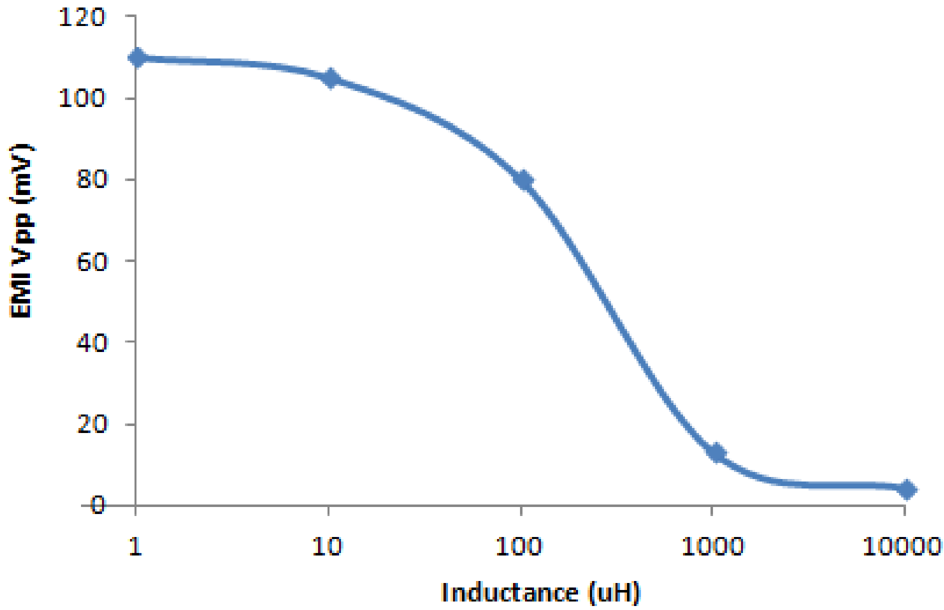


Figure 7.29: EMI as a function of series line inductor for the 10W load of Figure 7.46

As one can see, the largest drop of EMI conducted back to the input line occurs when the inductance value is in the vicinity of 10 mH or larger.

Design of 10 mH input line choke

We will use an MPP core in this design. Let's find McLyman's Area-Product to get the size we need:

$$Ap = [(2Energy)(1E4) / BmKuKj)]^{x}$$

From earlier:

$Ku = 0.4$

$Kj = 403$

$Bmax: = 0.3$ *Tesla*

$x = 1.14$

$L = 0.01\ H$

$i = 0.1\ A$

Therefore:

$Energy = 1/2L\ i^{2}$

$Energy = 5E\text{-}5$ *Joules*

Therefore the Area-Product is: $A_P = 0.012\ cm^4$

A look at the Magnetics catalog gives this core: 040 size (AP = 0.024 cm^4)

MMP core: <u>Magnetics 040 MMP core parameters:</u>

Ap	0.024 cm^4
Window area:	0.268 cm^2
Area of core:	0.0906 cm^2
Magnetic path length:	2.69 cm
Volume:	0.243 cm^3
Mass:	1.97 grams

With that settled we need to determine the number of turns we must wind on the core. This will require us to look at the *current density* which according to McLyman is:

$$J = Kj\ Ap^y$$

$$J = (403)(0.024)^{-0.12} = 629\ Amperes/cm^2$$

$$Awire_{(B)} = Imax / J$$

MMP core: $Awire_{(B)} = (0.1A)/(676A/cm^2) = 1.580E\text{-}4cm^2 \Rightarrow \#35\ AWG$

Information: #35 AWG *bare* copper cross sectional area of 0.1589E-3 cm^2.
#35 AWG *insulated* wire cross sectional area of 0.2268E-3 cm^2

Now that we know the wire size that will limit temperature rise to 25 °C (by virtue of the Kj parameter), we can find approximately how many turns of wire we need on the selected core. We need to use the insulated wire area values because that is the closest to real-world. Using this data we can determine the maximum number of turns we can place on each selected core:

$$N = Wa\ S_2 / Awire$$

where S_2 is a fill factor of 0.6 that takes into account the tightness of the winding.

MMP core: $N = (0.268)(0.6)/(0.0002268) = 708$ *turns #35 AWG*

This is a huge amount of turns to place on a toroid. We need to stop right here because winding this toroid would be cost prohibitive.

Would a ferrite pot core be any better? The bobbins for a ferrite pot core are much easier to wind. Let’s see what the Ap number would be:

$$Ap = [(2Energy)(1E4) / BmKuKj)]^{x}$$

Ku = 0.4
Bmax = 0.25
KJ = 433
x = 1.20

Ap needed = 0.011 cm^4

Chart 7-8 says that a 1408 pot core has an area product of 0.02 cm^4.

<u>1408 Core:</u>

Ap	0.0236 cm^4
Window area:	0.094 cm^2
Area of core:	0.251 cm^2
Magnetic path length:	1.98 cm
Length per turn:	2.89 cm
Volume:	0.495 cm^3
Mass:	3.2 grams

With that settled we need to determine the number of turns we must wind on each core. This will require us to look at the *current density* which according to McLyman is given by the following equation that allows a copper wire temperature rise of 25 °C:

$$J = Kj\ Ap^{y} \qquad (7\text{-}43)$$

We must insert the core data <u>of the core we will utilize</u> where we use K_J from the previous chart listing. Insert what corresponds to the core you have selected.

Ferrite pot core (1811 core size): $J = (433)(0.0236)^{-0.17} = 819\ Amperes/cm^2$

Because we know the maximum current of 0.10 Amperes will flow, we can calculate the <u>bare</u> wire size needed:

$Awire_{(B)} = Imax / J$

Ferrite pot core: $Awire_{(B)} = (0.10A)/(819A/cm^2) = 0.122E\text{-}3cm^2$ => #36 AWG

Our selection was based on the following information:

Information: #36 AWG bare copper cross sectional area of 0.127E-3 cm^2.
#36 AWG insulated wire cross sectional area of 0.181E-3 cm^2

Now that we know the wire size that will limit temperature rise to 25 °C (by virtue of the Kj parameter), we can find approximately how many turns of wire we need on the selected core. We need to use the insulated wire area values because that is the closest to real-world. Using this data we can determine the maximum number of turns we can place on each selected core:

$$N = Wa\, S_2 / Awire$$

where S_2 is a fill factor of 0.6 that takes into account the tightness of the winding.

Ferrite pot core: $N = (0.094)(0.6)/(0.000181) = 311$ *turns #36 AWG*

Now that we know the approximate number of turns that will fit and have the limited temperature rise, the required approximate A_L value can be determined.

$$A_L = L / N^2$$

Ferrite pot core: $A_L = (0.01\ H)/311^2 = 100\ nH / Turn^2$

The Ferroxcube catalog gives AL values for the 1408 core made of 3C81 material.

GRADE	A_L (nH)	μ_e	AIR GAP (µm)	TYPE NUMBER
3C81	63 ± 3%	≈ 40	≈ 680	P14/8-3C81-E63
	100 ± 3%	≈ 63	≈ 390	P14/8-3C81-A100
	160 ± 3%	≈ 100	≈ 220	P14/8-3C81-A160
	250 ± 3%	≈ 157	≈ 130	P14/8-3C81-A250
	315 ± 3%	≈ 198	≈ 100	P14/8-3C81-A315
	2800 ± 25%	≈ 1760	≈ 0	P14/8-3C81
3C91 des	2800 ± 25%	≈ 1760	≈ 0	P14/8-3C91

Figure 7.30: 1408 Pot core AL values

Looks like a total gap of 390 µm (0.015 inches) will allow us to have an exact A_L value equal to 100 nH/T^2. The effective gapped permeability as listed in the chart is 63. Winding a bobbin with 311 turns is much easier than winding a toroid. An insulating sheet of 7.5 mils is probably obtainable so this is what we will use – or we can purchase already pre-gapped cores. .

Design of filter chokes

Removing the ripple in a 60Hz brute-force high voltage power supply, is no easy task:

> **Example 8:** Design a filter for a Variac operated 5kV 600 Watt high voltage power supply in a 19 inch rack 3U high. The following schematic shows just such a topology:

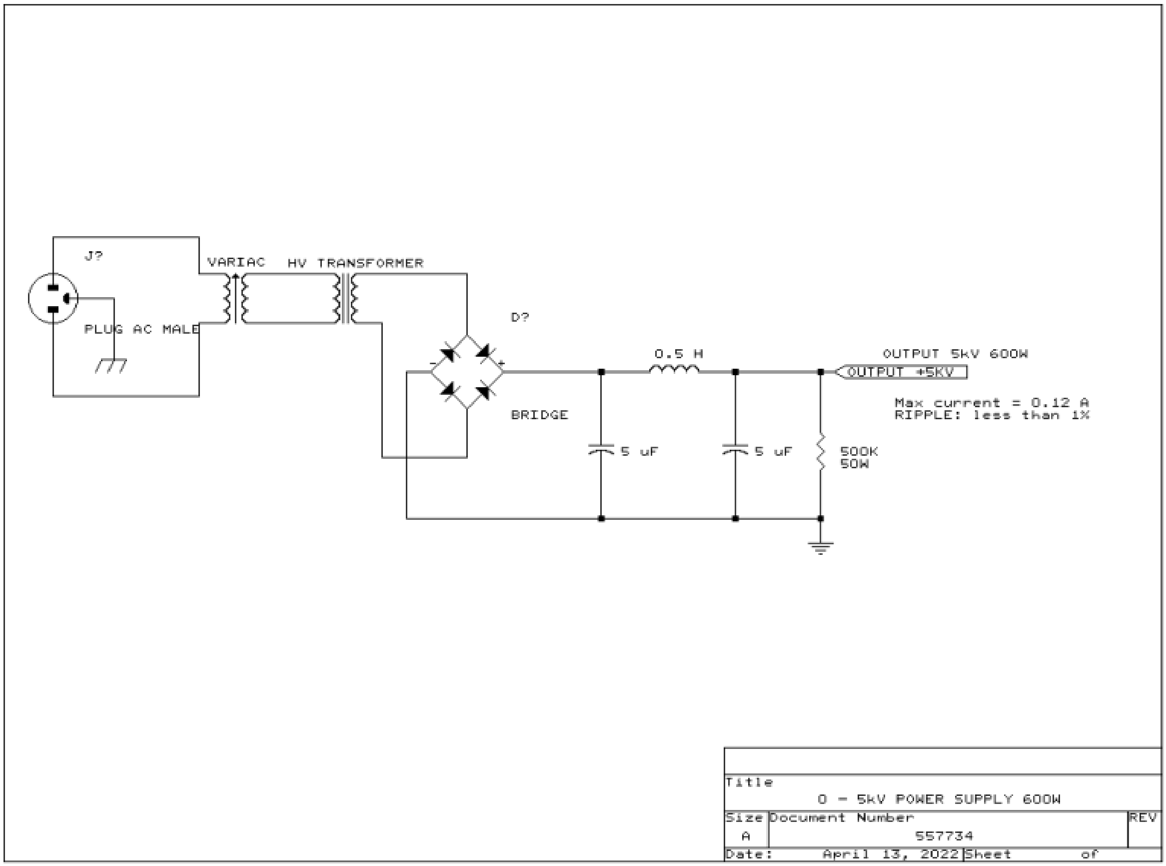


Figure 7.31: Five KV 600W supply

To keep the output ripple below 1%, (50V) and also keep minimum value on stored energy by not using too high filter capacitors, a pi-network choke of 0.5 Henrys will be used. This choke will use silicon steel laminations and must be able to handle 0.12 Amperes without much roll-off in inductance. Since this will fit within a 19 inch rack the size is not too important.

Table 7-8: Parameters used in the Area-Product analysis

	Bmax	Kj	x	y
Si-Fe Lamination:	1.2	366	1.14	-0.12

Si-Fe Lamination: $Ap = [114\ Energy]^{1.14}$

Energy = *½ Li*^{2}

If L is in Henrys and i in Amperes, the Energy term is in Joules. Because our L = 0.5 H and i peak = 0.12 Amperes we find the Energy term to be:

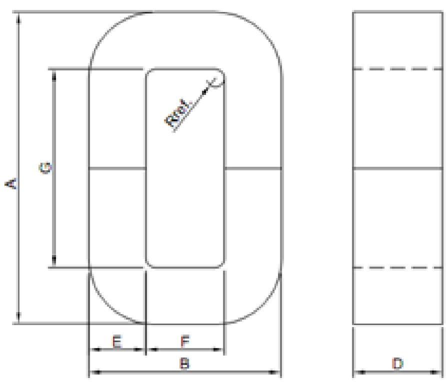
Energy = *0.0036* *Joules*

This is one spot where silicon steel laminations shine:

Ap = *0.362* *cm*^{4} *silicon steel*

Silicon steel C core:

Specifications of Standard C Cores:



Code	Dimension (mm)									Flux path	Cross section		Weight Kg	
	G	F	A	B	D	D	E	E	R	L(cm)	k (cm^2)		M	
	min	min	max	max	min	max	min	max	max	min	0.3mm	0.1mm	0.3mm	0.1mm
CD6.5×12.5×8	8.0	8.0	23.0	22.4	12.3	12.9	6.3	6.90	1.5	5.07		0.747		0.029

Figure 7.32: Silicon steel C core

<u>Nicore: CD6.5x12.5x8</u>

Ap	0.478 cm^4
Window area:	0.64 cm^2
Area of core:	0.747 cm^2
Magnetic path length:	5.07 cm
Volume:	4.14 cm^3
Mass:	29 grams
A_L	9257 nH/T^2 no gap
µe	5000

We have now selected the sizes of our core. Next, we need to determine the number of turns we must wind on our core. This will require us to look at the maximum *current density* which according to McLyman is given by the following equation that allows a copper wire temperature rise of 25 °C:

$$J = Kj\, Ap^{y}$$

We must insert the core data <u>of the core we will utilize</u> where we use K_J and y from Table 7-2. Insert what corresponds to the core selected.

Lamination core: $J = (534)(0.478)^{-0.12} = 583 \text{ Amperes/cm}^2$

This will limit the temperature rise in the copper to less than 25°C. Because we know the maximum current of 0.12 Amperes will flow, we can calculate the <u>bare</u> wire size needed: Calculating out the bare cross-sectional area needed gives us the wire size (when looking at the wire table in Appendix A in back of this book).

$$Awire_{(Bare)} = Imax / J$$

Lamination core: $Awire_{(Bare)} = (0.12A)/(583A/cm^2) = 0.2056E\text{-}3cm^2 \Rightarrow \#34$

Our selection was based on the following information:

Information: #34 AWG *bare* copper cross sectional area of 0.2011E-3 cm^2.
#34 AWG *insulated* wire cross sectional area of 0.2863E-3 cm^2.

Now that we know the wire size we can find approximately how many turns of wire we need on the selected core. We need to use the insulated wire area values because that is the closest to real-world.

Using this data we can determine the maximum number of turns we can place on the selected core:

$$N = Wa\, S_2 / Awire$$

where S_2 is a fill factor of 0.6 that takes into account the tightness of the winding.

Lamination core: $N = (0.64)(0.6)/(0.0002863) = 1341 \text{ turns \#34 AWG}$

Now that we know the approximate number of turns that will fit in the winding area of our core and generate a temperature rise less than 25 °C, the required approximate A_L value can be determined.

$$A_L = L / N^2$$

Lamination: $A_L = (0.5H) / 1341^2 = 278\ nH / Turn^2$

Determining the total gap:

We will use the "universal gapping equation" of Equation 7-35. Here is a case where we have to use it because we have no data as to the gaped A_L values so there is no TRENDLINE to plot. We only have a non-gapped value for this core from the manufacturer:

$$A_L = 9257\ nH/T^2$$

But we have our dimensions of the core.

$$\mu e_{gapped} = \mu_{e\ nogap} / (1 + \mu_{e\ nogap}\ lg / le))$$

with the "short" inductance equation:

$$L = \mu e_{gapped}\, \mu_o N^2 Ae / \ell e$$

$$0.5 = \mu e_{gapped} (4\pi E\text{-}7)(1341^2) (0.747E\text{-}4) / (0.0507)$$

giving: $\mu e_{gapped} = 150$

$$150 = 5000 / (1 + 5000(\ell g / 0.0507))$$

$$\ell g = 0.000327 \text{ meters}$$

$$\ell g = 0.327 \text{ millimeters (about 13 mils)}$$

Copper Losses

The McLyman analysis we are using limits the copper temperature rise to 25 degrees C. Nevertheless, we should calculate the copper wire loss for our inductor to make sure we don't have a potential fire hazard. All we have to do is determine the DC resistance for the winding and use:

$$Power = I^2R$$

To determine the loss we have used the maximum current value in our calculation of 0.12 Amperes.

	Turns	AWG	MLT (cm)	Wire length (cm)		Ω/cm (μ – Ω/cm)	R (Ohms)	loss (W)
	-------	-------	-------	--------		--------------	--------	------
Lamination core:	1341	34	5.76	7724	1687	8572	66	0.95

Core losses: Because the peak to peak voltage ripple across the inductor is 200 – 50 VAC, we can determine the maximum B field from Faraday's equation (equation 7-36):

$$Bmax = Vp / NAe 2\pi f$$
$$Bmax = (150) / (1341)((0.747E\text{-}4) 2\pi (120)$$

where we have used 120 Hz as the ripple frequency (full wave)

$$Bmax = 1.9 \ Tesla$$

From the Steinmetz equation of Chapter 6:

$$P_{hysteresis} = k_h f B_m^{1.4} V \qquad (7\text{-}44)$$

where P is the power loss in Watts, k is the Steinmetz coefficient (Chapter 6) for the material, f frequency, B_m the maximum value of B field in Tesla and V the volume of the ferromagnetic material in cubic meters. Silicon steel has a k_h value of 170 $W/m^3 \, T^2$ Hz.

$$P_{hysteresis} = (170)(120)(1.9)^{1.4}(4.14E\text{-}6)$$
$$P_{hysteresis} = 0.207 \ watts$$

we can neglect this loss, it is much lower than the copper loss.

Resonate Chokes:
Many times high voltage converters utilize a choke that converts pulse width modulated waves into amplitude varying sinusoidal waves which further drive step up transformers. At full output power they essentially transform a square wave into a sine wave by working in conjunction to any reflected capacitance from further circuitry. Some engineers say that the choke and capacitor combination acts as a sharply tuned filter – only passing the first harmonic towards the output load. This is not exactly correct because there is a greater voltage gain in the operation of the resonant than what would happen if we filtered out all of the higher order harmonics of a square wave. Detailed SPICE analysis of such a topology shows that whatever the description maybe, a simple duty cycle to voltage height converter can be easily fabricated to use in high

power switching converters that has several features such as high efficiency and friendly wave output with limited EMI spikes.

Example 8: Design a resonant power supply that takes 200 Vpp square waves at 40kHz and converts this into a high voltage output of 5,000 Vpp powering a 90 Watt resistive load. The winding capacitance and stray capacitance add to 20pF as seen by the secondary.

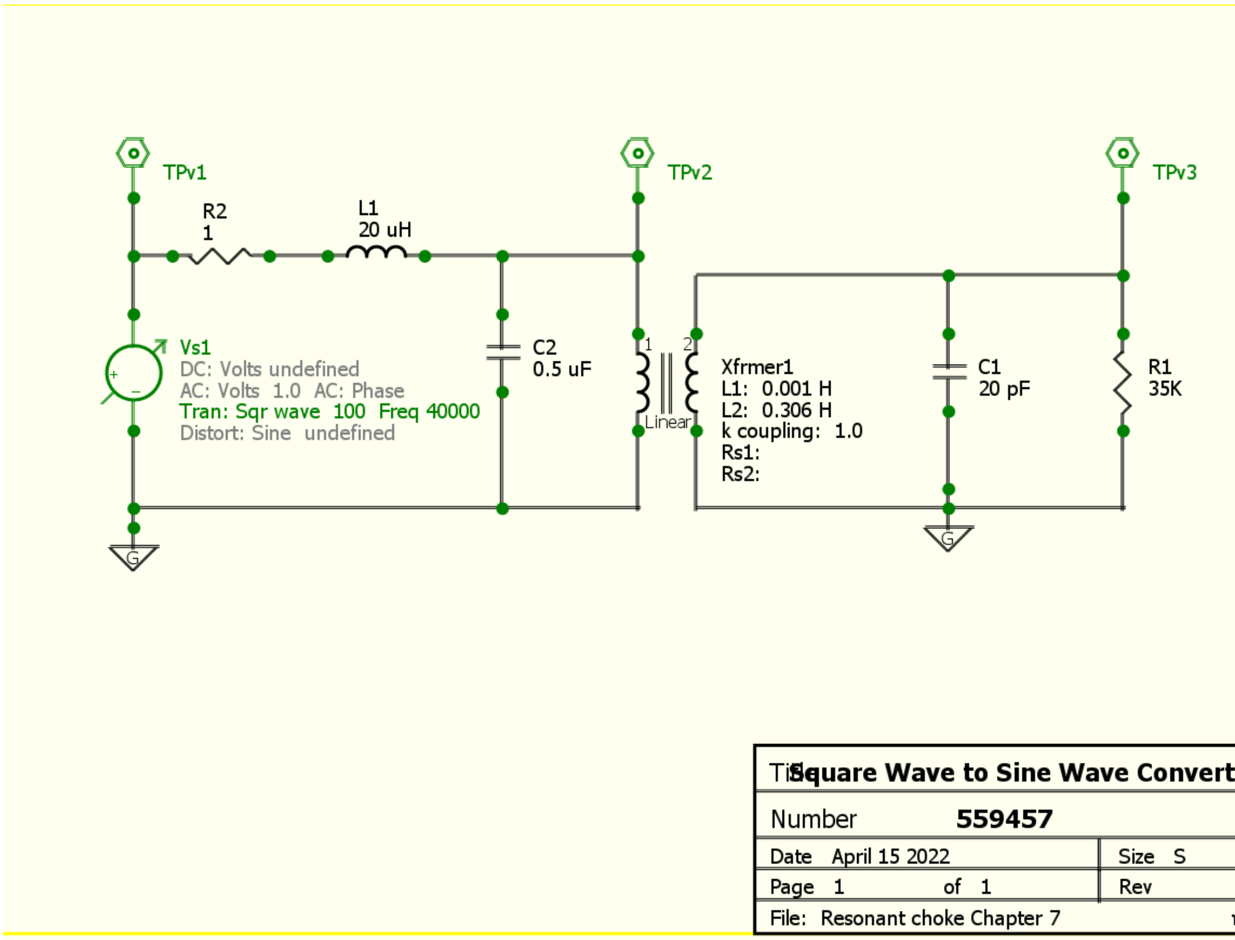


Figure: 7.33: Square wave to sine wave converter

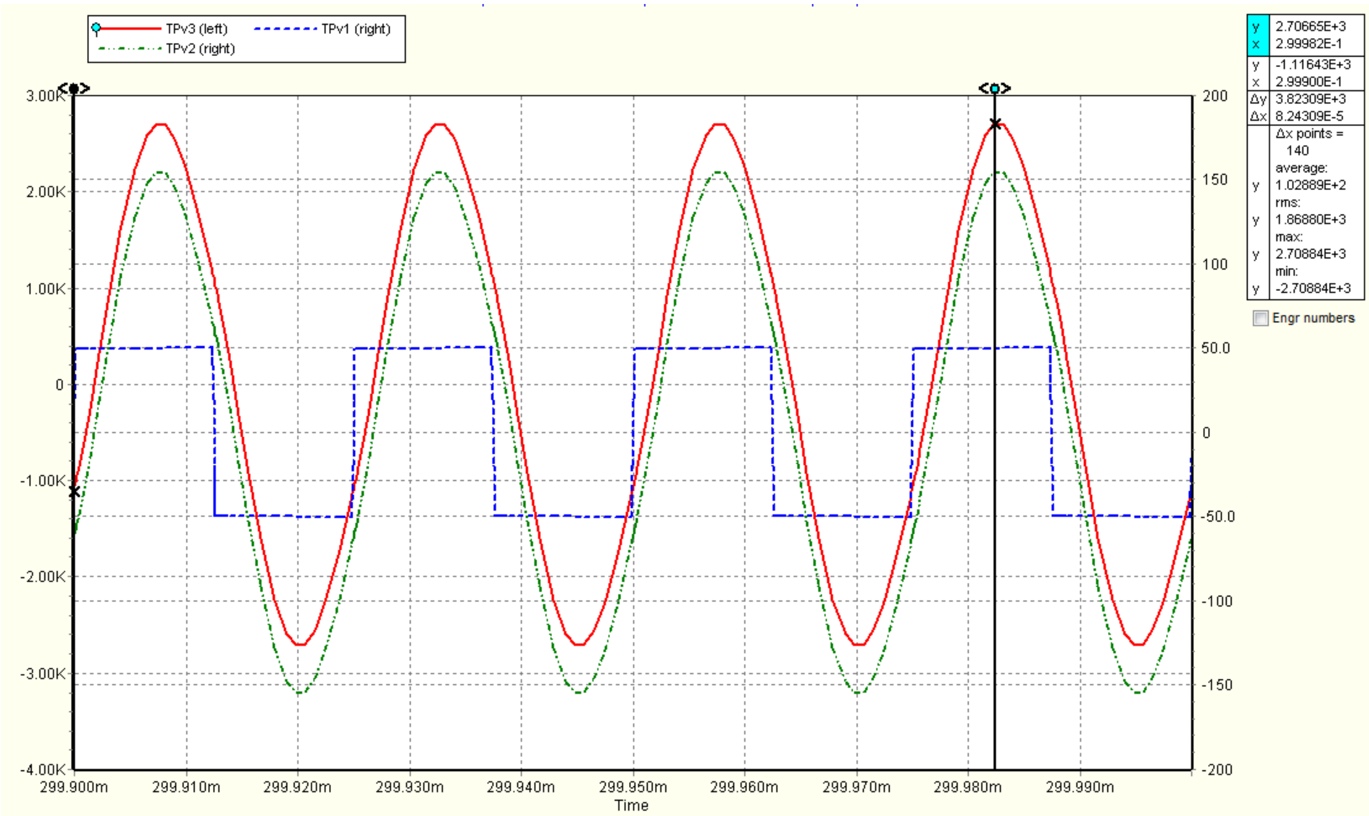


Figure: 7.34: Blue: Input of resonant inductor (right axis)
Green: Output of resonant inductor (right axis)
Red: Output of HV transformer 1:17.5 stepup (left axis)

Figure 7-33 shows the SPICE simulation circuitry for the power drive of a 90 Watt 5,000 volt output high voltage power supply operating at 40kHz driven from a square wave generator of amplitude 100 volts peak-peak. The step-up of the transformer is, dividing by the inductance ratio and taking the square root:

$$\text{Step-up} = \text{SQRT}(306) = 17.5.$$

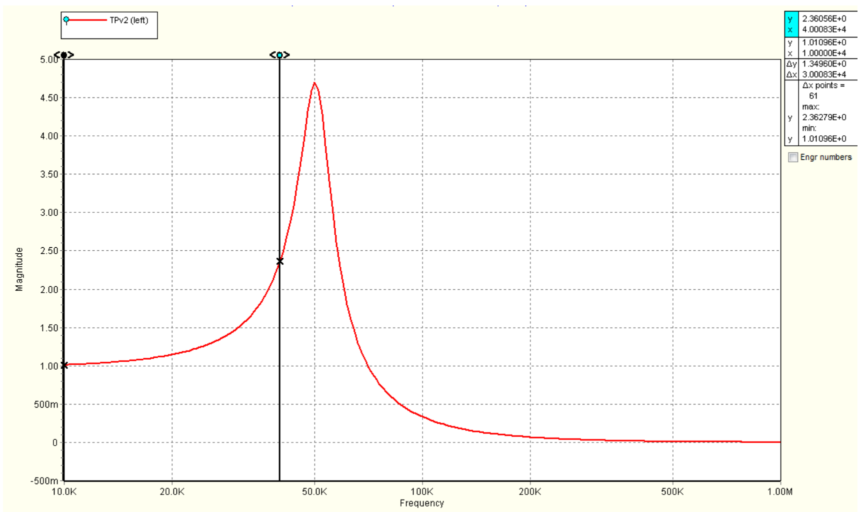


Figure 7.35: Gain VS frequency

Notice from Figure 7.33 that a large 0.5 uF capacitor has been placed in parallel across the primary of the transformer in order to make the resonant frequency be 50kHz, otherwise it is above 450kHz if this was not inserted.

$$fr = (1/2\pi)(\text{SQRT}(LC) \qquad (7\text{-}45)$$
$$fr = (1/2\pi)(\text{SQRT}((20\ uH)(0.506E\text{-}7))$$
$$fr = 50.0\ kHz.$$

Working at 40kHz, the gain from the actual waveforms taken from Figure 7.33 is:

$$Gain = Vtp2 / Vs1 = 155 / 50 = 3.1$$

It is important in these resonant circuits to always operate below the resonant frequency.

The output power into the 35K Ohm resistor is actually 104 Watts since the peak to peak waveform across this resistor from our simulation is 5400 volts.

A good question is: Will our transformer primary waveforms be sinusoidal? That can be deduced by calculating the "resonance resistance" of the series resonant circuit:

$$R_{res} = (1/2\pi)(1/SQRT\ (L / C) \quad (7\text{-}46)$$

$$R_{res} = (1/2\pi)(1/SQRT\ ((20\ uH) / (0.506\ uF)) = 6.28\ Ohms$$

As long as our load reflected back to the primary is greater than this number we will generate sine waves driving the step-up transformer because we are still in an under-damped situation which has sinusoidal solutions. When our reflected load of 114 Ohms, drops below 6.28 Ohms, that is if the actual output load drops from 35K to less than 1921 Ohms, we head into a critically damped case and our waveforms will be not a true sinusoid – they will be more trapezoidal in form. When this happens our resonant gain of 3.1 will drop to nearly unity.

Design of the 20 uH resonant inductor:

We will use a ferrite pot-core. The bobbins for a ferrite pot core are much easier to wind. Let's find what the Ap number would be and that we do by finding the current through the choke. Because the square wave come from a generator at 100 volts peak to peak, which is 50 volts RMS, you could think that the current would have to be 2 Amperes peak for a 100 Watt load and that is assuming 100% efficiency. Unfortunately, the large capacitor we inserted raises the current up ten-fold. The actual current flowing is 20 Amperes peak.

Therefore:

$$Energy = 1/2Li^2$$

$$Energy = (0.5)(20\ uH)(20^2)$$

$$Energy = 0.004\ Joules$$

Ferrite pot core: $Ap = [461\ Energy]^{1.20}$

Therefore the Area-Product is: $A_P = 2.08\ cm^4$

Figure 7.14 says that a 4229 pot core has an area product of 3.68 cm^4. We will use that one.

4229 Core:

Ap	3.68 cm^4
Window area:	1.40 cm^2
Area of core:	2.65 cm^2
Magnetic path length:	6.86 cm
Length per turn:	8.6 cm
Volume:	18.2 cm^3
Mass:	104 grams

With that settled we need to determine the number of turns we must wind on the core. This will require us to look at the *current density* which according to McLyman is given by the following equation that allows a copper wire temperature rise of 25 °C:

$$J = Kj\,Ap^{y}$$

We must insert the core data <u>of the core we will utilize</u> where we use K_J from the previous chart listing. Insert what corresponds to the core you have selected.

Ferrite pot core (4229 core size): $J = (433)(3.68)^{-0.17} = 347\ Amperes/cm^2$

Because we know the maximum current of 20 Amperes will flow, we can calculate the <u>bare</u> wire size needed:

$$Awire_{(B)} = Imax / J$$

Ferrite pot core: $Awire_{(B)} = (20A) / (347\ A/cm^2)$

$$Awire_{(B)} = 0.058\ cm^2 \Rightarrow 11\ strands\ \#20\ AWG$$

$$Awire_{(B)} = (11)(5.188E\text{-}3) = 0.057\ cm^2$$

Our selection was based on the following information:

Information: one strand #20 AWG *bare* copper cross sectional area of 5.188E-3 cm^2.
one strand #20 *insulated* wire cross sectional area of 6.065E-3 cm^2
eleven strands #20 *insulated* wire cross sectional area of 0.0667 cm^2

Using eleven strands of wire not only makes it easier to wind but reduces the AC loss due to skin effect. We are in a sense making our own Litz wire.

Now that we know the wire size that will limit temperature rise to 25 °C (by virtue of the Kj parameter), we can find approximately how many turns of wire we need on the selected core. We need to use the insulated wire area values because that is the closest to real-world. Using this data we can determine the maximum number of turns we can place on each selected core:

$$N = Wa\ S_2 / Awire$$

where S_2 is a fill factor of 0.6 that takes into account the tightness of the winding.

Ferrite pot core: $N = (1.40)(0.6) / (0.0667) =$ *13 turns 11x #20AWG*

Rounding off to 13 turns we have the approximate number of turns that will fit and have the limited temperature rise, the required approximate A_L value can be determined.

$$A_L = L / N^2$$

Ferrite pot core: $A_L = (20\ \mu H) / 13^2 = 118\ nH / Turn^2$

The Ferroxcube catalog gives AL values for the 4229 core made of 3C81 material.

GRADE	A_L (nH)	μ_e	TOTAL AIR GAP (µm)	TYPE NUMBER
3C81	315 ±3%	≈ 65	≈ 1320	P42/29-3C81-E315
	400 ±3%	≈ 82	≈ 990	P42/29-3C81-E400
	630 ±3%	≈ 130	≈ 580	P42/29-3C81-A630
	1000 ±3%	≈ 206	≈ 340	P42/29-3C81-A1000
	1600 ±5%	≈ 330	≈ 190	P42/29-3C81-A1600
	11500 ±25%	≈ 2370	≈ 0	P42/29-3C81

Figure 7.36: 4229 Pot core AL values

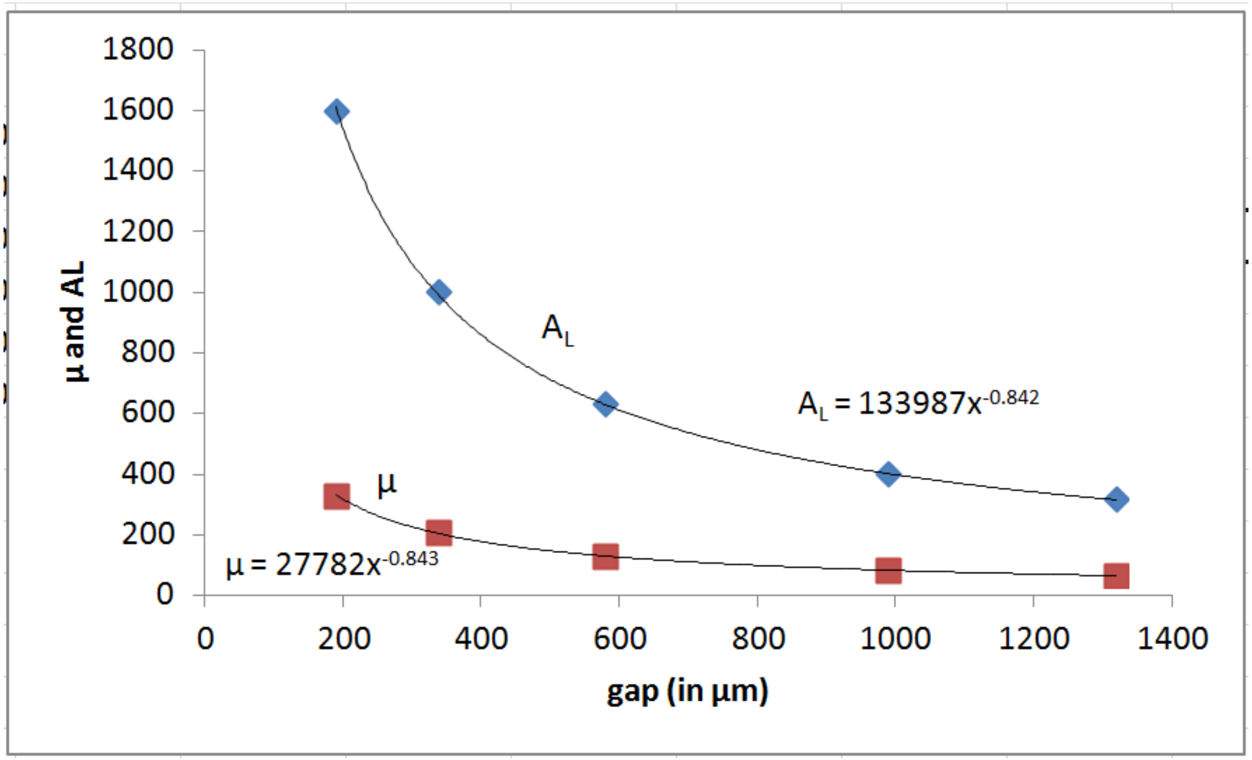


Figure 7.37: TRENDLINE of 4229 Pot core AL values and effective permeability

We need an A_L value of 118 nH/T^2 so the gap has to be larger than 1320 μm. Using logs we can determine its absolute value:. Using the TRENDLINE for A_L:

$A_L(in\ nH/T^2)$	=	$133987\ x^{-0.842}$
118	=	$133987\ (gap\ in\ \mu m)^{-0.842}$
$\log_{10}(118)$	=	$\log_{10}(133987)\ -\ 0.842\ \log_{10}(gap\ in\ \mu m)$
2.072	=	$5.127\ -\ 0.842\ \log_{10}(gap\ in\ \mu m)$
$4250\ \mu m$	=	gap

Roughly 5/32 inch for a total gap. Since there are no cores available from Ferroxcube that have their inner post ground to that amount, and we really cannot grind our own, a 5/64 shim of plastic such as G-10 can work for both the inner and outer gaps. We will standardize with a 3968 μm gap (5/32 inches - total). This would yield an A_L value of:

$A_L(in\ nH/T^2)$	=	$133987\ x^{-0.842}$
$A_L(in\ nH/T^2)$	=	$133987\ (3968)^{-0.842}$
A_L	=	$125\ \ nH/T^2$

The number of turns needed is thus:

$$N = SQRT\ (L / A_L)$$
$$N = SQRT\ (20\ \mu H / 125E\text{-}9)$$
$$N = 12.6\ turns$$

we will round to 13 turns. Using our gap of 5/32 inches, (3968 μm), the effective permeability is 25.7. So we will wind our inductor of 13 turns of 11 strands of #20 AWG insulated wire, the strands twisted together at 3 turns per inch. Because we have used the data from the catalog we can expect that the inductance will be pretty close despite the rather large gap.

Copper Losses

The McLyman analysis we are using limits the copper temperature rise to 25 degrees C. Nevertheless, we should calculate the copper wire loss for our pot core input line choke just in case there is a problem. The power loss is based on the I^2R resistive loss. To determine the loss we have used the maximum current value in our calculation the RMS value of 25 peak Amperes which is: 17.6 Amperes since our current waveform is a sinewave.

	Core	Turns	AWG	MLT (cm/T)	Wire length (cm)	Ω/cm (μ – Ω)	R (Ohms)	loss (W)
	-------	-------	-------	------	--------------	--------	-------------	------
Ferrite pot core:	4229	13	11#20	8.6	111.8 cm	332E-6	0.0034*	1.35

* This is the resistance value for all 11 strands in parallel.

4229 Ferrite pot core loss:

Because we are driving our resonant inductor with a maximum voltage of 150 V having 13 turns, we can determine our B field using Faraday's law as before:

$$Bmax = Vp / NAe 2\pi f$$
$$Bmax = (150) / (13)((2.65E\text{-}4) 2\pi (40{,}000)$$

where we have used 120 Hz as the ripple frequency (full wave)

$$Bmax = 0.17\ Tesla$$

Core loss Chart:

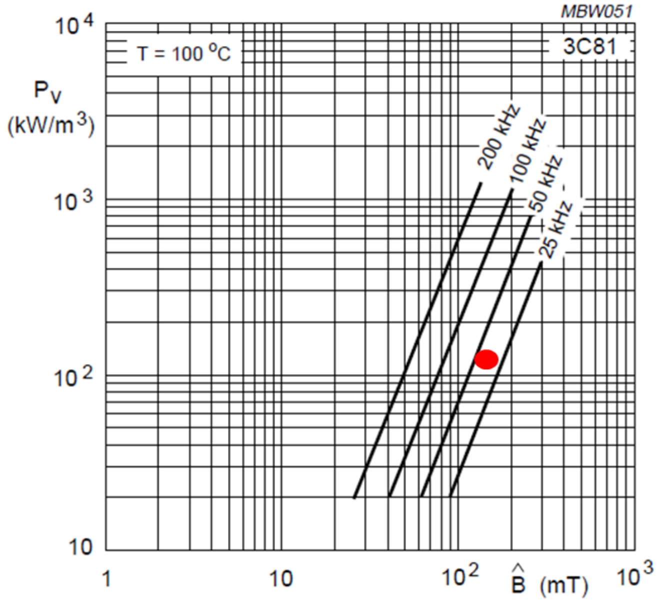


Figure 7.38: Resonant choke core loss 3C81

From Ferroxcube:

	B_{ACpeak}	loss (from chart)	volume	loss
	-------	---------------	--------------	--------
3C81 core loss:	0.15 Tesla	130 mW/cm^3	18.2 cm^3	2.34 W

Summation of losses for our ferrite resonant choke:

		Copper loss	Core loss	Sum of loss
		------------------	-------------	---------------
1408	Ferrite pot core:	1.35 W	2.34 Watts	3.72 Watts

For a 50°C rise in temperature a simple rule of thumb is to use the following equation:

$$Power_{50} \quad = \quad 2500\ a\ b \qquad (7\text{-}47)$$

where a and b are two of the largest dimensions in meters. For a 4229 pot core a = 42 mm and b = 29 mm. Inserting and converting to meters, we have:

$$Power_{50} \quad = \quad (2500)(0.042)(0.029) \quad = \quad 3.05 \text{ Watts}$$

Since out pot core dissipates more than 3.05 Watts, we would expect a temperature rise of:

Tempeature = (50)(3.72 / (3.05) = 61 °C

Drop in Inductance

How much will the inductance drop when we use this core?

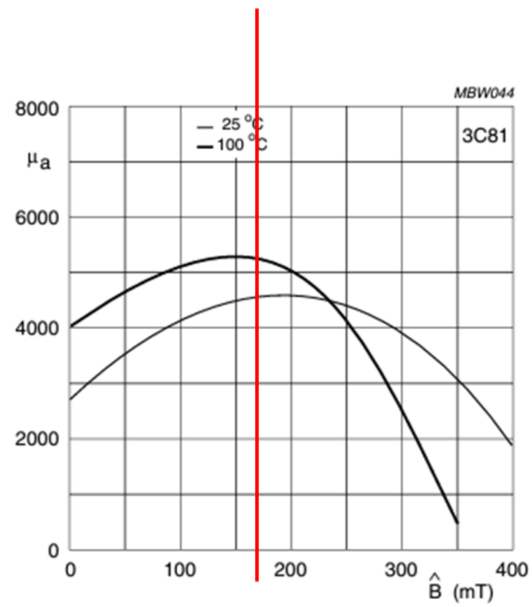


Fig.4 Amplitude permeability as a function of peak flux density.

Figure 7.39: Droop in μ due to B field

Figure 7.37 shows that the permeability holds steady for B fields at least up to 0.2 Tesla so we can say that the inductance of 20 uH probably won't change much over the range of low output to full output conditions. This chart was taken from the Ferroxcube data sheet for 3C81 material.